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Competition, Human Capital and
Income Inequality with Limited Commitment

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1 Introduction

Human capital accumulation plays an important role in the mechanics of economic growth as a complementary factor to physical capital, technological innovations and, with knowledge spillovers, to human capital itself. In turn, economic growth stimulates the accumulation of human capital by raising its return. Such bidirectional effects are at the core of growth theories based on endogenous human capital accumulation (e.g. Nelson & Phelps (1996) and Lucas (2002)). In these theories, higher competitive wages are the usual channel through which human capital is rewarded.

In this paper we focus on human capital accumulation as a key to technological innovation, either in the adoption of existing technologies or in the development of new technologies. We use a dynamic general equilibrium model where contracts are not enforceable, neither for workers nor for firms. The limited commitment of workers means that they can always quit the firm. The limited commitment of firms means that they can renegotiate the payments promised to the workers after the investment in human capital has been made. These contractual frictions affect the accumulation of both human and physical capital.

One contribution of this paper is to show that the way limited enforcement of contracts affects the accumulation of human capital depends on barriers to the mobility of skilled labor. In particular, we show that high barriers discourage the accumulation of human capital while low barriers have a stimulating effect. As a result, differences in ‘barriers to competition’ translate into significant differences in incomes and welfare across economies.

The key mechanism through which barriers to the mobility of skilled labor affect the accumulation of human capital is to reduce the outside option of skilled workers, that is, the value of redeploying their skills outside the firm. With a lower outside value, the worker does not have a credible mechanism for punishing the firm in case of renegotiation of the promised payments. Anticipating this, the worker does not have an incentive to provide the effort to acquire the skills. This is a typical time-inconsistency problem.

The limited commitment from the worker, in addition to the limited commitment from the firm (double-sided), is important for this result. If the worker could commit to staying with the firm and providing effort (one-sided commitment from the worker) the contractual friction could be resolved by making advance payments to the worker. However, without this commitment, advance payments are not incentive-compatible because the worker

could quit after being paid.

In line with existing Contract Theory, one can try to solve the commitment problem with an output-sharing agreement or by transferring total or partial ownership of assets to the workers (e.g. Hart & Moore (1994)). But with two-sided lack of commitment, such arrangements are still open to unverifiable *de facto* renegotiations (or skimming).

The time-consistent solution is such that the worker invests in human capital up to the point where there is no discrepancy between *ex-ante* promised payments to the workers and *ex-post* outside values (given the accumulated human capital). In our economy, the best outside option for a skilled-worker is to enter into a contractual arrangement with a new firm. Therefore, a credible investment policy for an incumbent firm is to mimic the investment decisions of a new firm. However, when the investment cost of new firms is high, that is, there are high barriers, their investment is low. Consequently, investment by incumbent firms is also low. In contrast, with full or one-sided commitment, incumbent firms do not mimic the investment decisions of new firms, and only the latter are directly affected by start-up costs.

Our results are first illustrated with a simple two-stage model which is then extended to a dynamic infinite horizon set-up. The parametrization of the infinite horizon model allows us to quantify the ability of one particular barrier—*start-up costs*—to account for different levels of human capital accumulation and innovation, as well as cross-country income differences. The baseline model accounts, roughly, for half of the cross-country income gaps with the US. Even though this number should be taken with caution, given the simplicity of the model, it shows that this mechanism can be quantitatively important, bringing a new perspective on the role of competition as a factor of growth. We deliberately use a semi-endogenous growth model (Jones (1995)) to explain income differences, as opposed to long-run growth differences, since this is what the evidence on the potential role of *start-up costs* suggests. We discuss this evidence in Section 2.

Our results can also be interpreted as saying that *barriers* to competition determine cross-country positions relative to the ‘technology possibility frontier’, without emphasizing a distinction between innovation and technology adoption, which is consistent with the idea that even the implementation of known technologies requires appropriate human capital. In contrast, Acemoglu, Aghion, & Zilibotti (2006) take the ‘technology possibility frontier’ as given and develop a theory where the ‘distance to the frontier’ determines a country’s comparative advantage on innovation vs. adoption. Moreover,

while in their theory the cost of barriers depends on the position of a country relative to the frontier, in our framework it is the barriers that determine the position of a country in relation to the frontier. The causality effect is reversed and the policy implications are very different. They argue the lack of pro-competitive policies becomes more costly as countries approach the world technology frontier, while our theory implies that the lack of pro-competitive policies can determine a country's position away from the frontier, as our computations of the dynamical model show.

We also show how other barriers to mobility such as *covenants* (preventing a skilled-worker from working for a period in the same industry), can be incorporated in our model to account for regional differences. For example, the evolution of the computer industry exemplifies the effects of both types of *barriers* to competition. As Bresnahan & Malerba (2002) emphasize, this industry has gone through different technological stages (from main-frames to PCs and the Internet). Knowledge in this particular industry was geographically spread in many countries including Europe. Yet the United States has persistently been the industry leader. According to them, this dominance can be explained by “...*the existence of a large body of technical expertise in universities and the generally supportive environment for new firm formation in the United States*”, Bresnahan & Malerba (2002, page 69).

While lower barriers to business start-up may have favored the computer leadership of the United States, different enforcement of covenants—and informational linkages across firms—may have determined the shift of regional leadership within the United States. As argued by Saxenian (1996), Gilson (1999) and Hyde (2003), Silicon Valley dominates over Route 128 due to a Californian legal and social tradition of not enforcing post-employment covenants, resulting in high labor mobility and knowledge spillovers.

This paper relates to different strands of literature. In addition to the ones already cited, at least two more should be mentioned. First, the labor literature that studies the accumulation of skills within the firm (e.g. Acemoglu (1997), Acemoglu & Pischke (1999), Acemoglu & Shimer (1999)). In this literature, higher outside values worsen the hold-up problem and lead to lower accumulation of skills. In our framework, instead, higher outside values increase human capital investment. Second, by emphasizing the role of barriers to mobility, our work also relates to the literature that emphasizes the role of barriers to riches in slowing growth and explaining income differences (Mokyr (1990) and Parente & Prescott (2002)).

2 Cross-country evidence on barriers to business start-up

Before describing the theoretical framework, we present here some cross-country data suggesting a relation between the cost of business start-up—which in our theory is one of the barriers to knowledge mobility—and cross-country income. It is important to emphasize that our theory is broader than simply capturing the impact of barriers to business start-up. Here, and in the later application of the theory, we will focus on barriers to business start-up only because this is the data we have available.

A recent publication from the World Bank (2005) provides indicators of the quality of the business environment for a cross-section of countries. It also includes proxies for barriers to business start-up. There are three main variables. The first is the ‘cost of starting a new business’. This is the average pecuniary cost needed to set up a corporation in the country, in percentage of the country per-capita income.¹ The second proxy is the ‘number of bureaucratic procedures’ that need to be filed before starting a new business. The third proxy is the average ‘length of time’ required to start a new business.

Figure 1 plots the level of per-capita GDP in 2004 against these three indicators, where all variables are in log. All panels show a strong negative correlation, indicating that the set-up of a new business is more costly and cumbersome in poor countries.

The cost of business start-up is also negatively correlated with economic growth. To show this, we regress the average growth in per-capita GDP from 2000 to 2004 to the cost of business start-up. We also include the 1999 per-capita GDP to control for the initial level of development. We would like to emphasize that the goal of these regressions is not to establish a causation but only to highlight some key correlations that motivate our study. The estimation results, with *t*-statistics in parenthesis, are reported in Table 1.

As can be seen from the table, the cost of business start-up is negatively associated with growth even if we control for the level of economic development. Therefore, countries with lower barriers to entry tend to experience faster growth. This finding is robust to the choice of alternative years for

¹The normalization of the cost of business start-up by the level of per-capita income better captures the importance of barriers to business start-up than the absolute dollar cost. What is relevant is the comparison between the cost of business start-up and the value of creating a business. Although the dollar cost is on average higher in advanced economies, the value of a new business is also likely to be higher.

Figure 1: Barriers to business start-up and level of development.

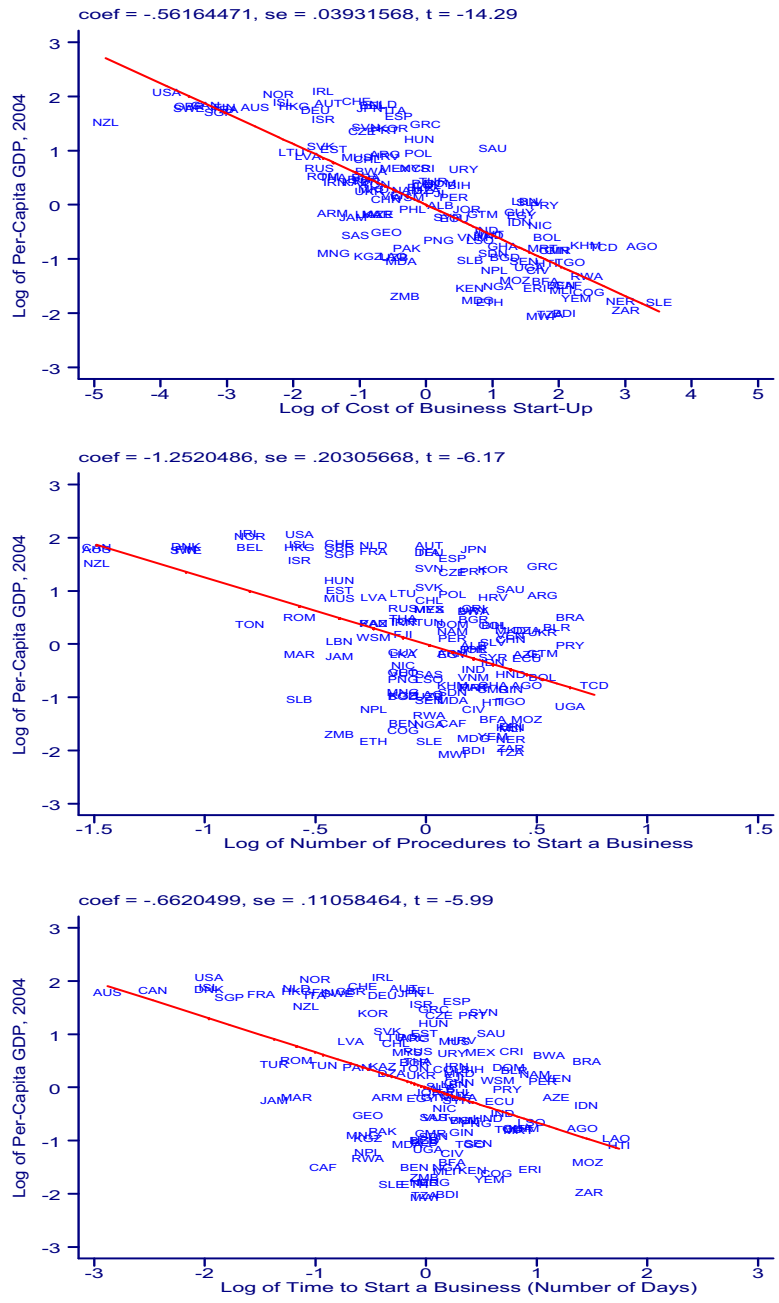


Table 1: Cost of business start-up and growth.

	<i>Constant</i>	<i>Initial Per-Capita GDP</i>	<i>Cost of Business Start-Up</i>
Coefficients	15.55	-1.16	-1.04
<i>t</i> -Statistics	(5.01)	(-3.81)	(-4.92)
<i>R</i> -square	0.150		
<i>N. of countries</i>	140		

NOTES: The dependent variable is the average annual growth rate in per-capita GDP for the five year period 2000-2004. Initial Per-Capita GDP is the log of per-capita GDP in 1999. The cost of business start-up is in percentage of the per-capital Gross National Income as reported in *Doing Business in 2005* (also in log)

compute the average growth rate. The other proxies for barriers to entry—specifically, the number of procedures and the time required to start a new business—are also negatively correlated with growth but they are not statistically significant at conventional levels.

To summarize, the general picture portrayed by the data is that economic development and growth is negatively associated with the cost of starting a business. We have presented simple correlations which, of course, do not imply causation. In the following section we present a model where barriers to entry and, more generally, barriers to the mobility of knowledge or human capital, lead to lower income and growth. We will return to the cross country data presented here in the quantitative analysis of Section 6.

3 The model

There is a continuum of ‘workers’, of total mass 1, each characterized by a level of human or knowledge capital h_t . Their lifetime utility is $\sum_{t=0}^{\infty} \beta^t (c_t - e_t)$, where c_t is consumption and e_t is the ‘effort’ to accumulate knowledge as specified below. In addition to workers there is a continuum of ‘investors’ of total mass $m > 1$. Investors are risk neutral with lifetime utility $\sum_{t=0}^{\infty} \beta^t c_t$. The risk neutrality implies that the equilibrium interest rate is equal to the intertemporal discount rate; that is, $r = 1/\beta - 1$.

Production projects require the input of knowledge capital, h_t , and phys-

ical capital, k_t . They generate output according to:

$$f(k_t, h_t) = h_t^{1-\alpha} k_t^\alpha.$$

We assume that workers do not save, and therefore, physical capital must be provided by investors. The goal of this assumption is to differentiate the roles of workers and investors: the first as providers of human capital and the second of financial resources. If workers saved, they would be able to self-finance the purchase of physical capital eliminating the contractual frictions between investors and workers.²

Investors compete to hire workers in a Walrasian market by offering contracts that determine the investment in human and physical capital and the compensation structure. We refer to the contractual arrangement between an investor and a worker as a *firm*. For expositional simplicity we assume that each investor can hire only one worker. However, the investor-worker pair can also be interpreted as a specific project or unit within a large firm with certain common features. First, the relationship with each worker is governed by a specific contract; second, investors behave competitively (for example, they can not collude to prevent workers' mobility); third, workers are in the short side of the market; that is, the ability to hire workers exceeds the number of workers. In the model this is obtained by assuming that the mass of investors is bigger than the number of workers ($m > 1$). Essentially, this is equivalent to the free entry assumption in many models of the labor market including those with matching frictions.³

As anticipated above, investment in knowledge or human capital, $h_{t+1} - h_t$, requires effort from the worker. The effort cost function is denoted by:

$$e_t = \varphi(h_t, h_{t+1}; H_t),$$

where H_t is the economy-wide knowledge. The dependence on the aggregate knowledge captures possible leakage or spillover effects.

The function φ is strictly decreasing in H_t and h_t , strictly increasing and convex in h_{t+1} , and satisfies $\varphi(h_t, h_t; H_t) > 0$. It is further assumed to be

²Zero savings could also be interpreted as an endogenous outcome if workers discount more heavily than investors. As long as the discount differential is sufficiently high, workers will not save in equilibrium.

³As we will see, barriers to entry are not in the form of restrictions on the number of entrants but burdens imposed on new firms. Therefore, we keep the assumption of free entry.

homogeneous of degree $\rho > 1$. With this homogeneity assumption the model generates only long-term differences in income *levels*, and therefore, this is a semi-endogenous growth model as in Jones (1995). The analysis can be easily extended to $\rho = 1$, in which case we would have long-term *growth* differences.⁴

Physical capital is knowledge-specific. When a worker upgrades the level of knowledge, only part of the existing capital is usable with the new knowledge. Knowledge upgrading is equivalent to the adoption of a new technology that makes part of the existing equipment obsolete. Capital obsolescence increases with the degree of knowledge upgrading. This is formalized by assuming that the depreciation rate of physical capital is:

$$\delta_t = \delta \cdot \left(\frac{h_{t+1}}{h_t} \right).$$

Because of physical capital obsolescence, there is an asymmetry between *incumbent firms*—whose capital depreciates with more advanced knowledge—and *new firms* which, without capital in place, have a greater incentive to hire workers with higher knowledge (Arrow’s ‘replacement effect’).

Competitive structure and barriers: In each period there is a mass 1 of investors who are in a contractual relationship with workers, and a mass $m - 1 > 0$ who are idle and could start new firms. Investors can borrow from and lend to each other to finance the capital k at the interest rate r . The labor market is competitive and opens twice, before and after the accumulation of knowledge. Potential new firms funded by idle investors create a competitive demand for workers (human capital) and physical capital. Even though there is no entry in equilibrium, the potential for the creation of new firms is crucial for the characterization of the equilibrium.

The effective competition for workers (and physical capital) created by potential entrants is limited by several types of barriers. For the moment, we consider only barriers to business start-up. The analysis of other barriers, such as the strict enforcement of covenants, will be conducted in Section 7 with similar results.

⁴The model can be interpreted as a detrended version of an economy that grows at the exogenous rate dictated by worldwide knowledge. Let \bar{H}_t be worldwide knowledge growing at the exogenous rate \bar{g} , with the effort cost function, $e_t = \tilde{\varphi}(h_t, h_{t+1}; H_t, \bar{H}_t)$, homogeneous of degree 1. After normalizing all variables by \bar{H}_t , the cost can be written as $\varphi(h_t, h_{t+1}; H_t)$, which is homogeneous of degree $\rho > 1$.

Barriers to entry are modeled as a deadweight cost proportional to the initial level of knowledge chosen by the firm. Given the initial knowledge h_{t+1} , the entry cost is $\tau \cdot h_{t+1}$. The key results of the paper are robust to alternative specifications of the entry cost. Our choice is only motivated by its analytical convenience.⁵

4 One-period model

Before studying the general model with infinitely lived agents, we first consider a simplified version with only one period to facilitate the intuition for the key results of the paper. The analysis with an infinite horizon, however, is still important because it allows us to derive the initial conditions endogenously as steady state values and, more importantly, it allows us to explore the model quantitatively in Section 6.

There are two stages: before and after the investment in knowledge. The states at the beginning of the period are h_0 and k_0 . After making the investment decisions, h_1 and k_1 , the firm generates output $y_1 = h_1^{1-\alpha} k_1^\alpha$ in the second stage. In this simple version of the model we assume that physical capital fully depreciates after production. The worker receives a payment w at the end of the period, i.e. after the choice of h_1 . Payments before the choice of h_1 are not incentive-compatible because of the limited enforcement of contracts for the worker. With only one period, we can ignore discounting, as well as leakage or spillover effects.

The timing of the model is as follows: The firm starts with initial states h_0 and k_0 . At this stage the worker decides whether to stay or quit the firm. If the worker quits, she can be hired by a new firm (funded by a new investor). The capital owned by the incumbent firm, k_0 , can be sold to the new firm at a price we derive below. If the worker decides to stay, she will exercise effort to upgrade the knowledge capital to h_1 and the investor provides the funds to upgrade the physical capital to k_1 . After the investment, the firm pays w . At this stage the worker can still quit and switch to a new firm. However, she cannot change the level of knowledge h_1 . The investor is the residual claimant of the firm's output.

⁵For example, we could assume that the cost is proportional to the initial capital k_{t+1} , or to the initial output $h_{t+1}^{1-\alpha} k_{t+1}^\alpha$, or to the discounted flows of outputs. The basic theory and results also apply when the entry cost is a fixed payment. The assumption of proportionality allows for a continuous impact of τ while a fixed cost would have an impact only after it has reached the prohibitive level.

4.1 Some preliminaries

Negotiation takes the form of Nash bargaining between the incumbent firm and the worker. To solve for the contracting problem we need to define the surpluses and outside options for both, the worker and the firm.

The surpluses for the worker, before and after the knowledge investment are:

$$W(h_0, k_0) = w - \varphi(h_0, h_1) \quad (1)$$

$$\widehat{W}(h_1, k_1) = \widehat{w}, \quad (2)$$

and for the firm:

$$J(k_0, h_0) = -w - k_1 + \left[1 - \delta \cdot \left(\frac{h_1}{h_0}\right)\right] k_0 + h_1^{1-\alpha} k_1^\alpha \quad (3)$$

$$\widehat{J}(h_1, k_1) = -\widehat{w} + h_1^{1-\alpha} k_1^\alpha. \quad (4)$$

From now on we will use the *hat* sign to denote the functions that are defined *after* the investment in knowledge (second stage).

The repudiation values or outside options for the worker are the values of quitting the current employer and switching to a new firm. Because of competition among potential entrants, these are the surpluses created by new firms, which are given by:

$$D_w(h_0) = \max_{h_1, k_1} \left\{ -\varphi(h_0, h_1) - \tau h_1 - k_1 + h_1^{1-\alpha} k_1^\alpha \right\} \quad (5)$$

$$\widehat{D}_w(h_1) = \max_{k_1} \left\{ -\tau h_1 - k_1 + h_1^{1-\alpha} k_1^\alpha \right\} \quad (6)$$

where again we have used the hat sign to denote the repudiation value after the investment in knowledge.

The assumption that there is competition among potential investors and workers get the whole surplus created by new firms is without loss of generality. In Appendix A we will consider the more general case in which workers get a share of the surplus and obtain the same results.

New firms create a competitive demand also for the physical capital of incumbent firms. The purchase of physical capital from an existing firm is equivalent to the acquisition of that firm and, through this, the new firm

avoids the payment of the entry cost τh_1 . Therefore, the prices that new firms are willing to pay for acquiring the capital of an incumbent firm are:

$$D_f(k_0, h_0) = \left[1 - \delta \cdot \left(\frac{h_1^{New}}{h_0} \right) \right] k_0 + \tau h_1^{New} \quad (7)$$

$$\widehat{D}_f(k_1, h_1) = k_1 + \tau h_1 \quad (8)$$

where h_1^{New} is the knowledge investment chosen by a new firm created before the investment in knowledge.

In other words, new firms are willing to pay a price that is equal to the cost they save by purchasing the firm. At the beginning of the period this is equal to the value of the old capital k_0 given the planned investment h_1^{New} and the entry cost τh_1^{New} . After the investment in knowledge, the physical capital chosen by a new firm is exactly equal to the capital owned by the incumbent firm.

4.2 Equilibrium with one-sided limited commitment

We first characterize the equilibrium when at least one of the parties, either the investor or the worker, commits to the contract. In the context of the one period model, commitment implies that the agent does not renegotiate the contract in the second stage of the period, after the investment in knowledge.

We start with the characterization of the equilibrium when only the investor commits. It will then be trivial to show that the equilibrium investment does not change when the worker commits. As we will see, it is the limited commitment of both parties (double-sided limited enforcement) that induces a deviation from the optimal investment.

With investor's commitment, the optimal contract can be characterized by choosing all variables at the beginning of the period through Nash bargaining. The optimal policy solves:

$$\max_{h_1, k_1, w} \left[W(h_0, k_0) - D_w(h_0) \right]^{1-\eta} \left[J(h_0, k_0) - D_f(h_0, k_0) \right]^\eta \quad (9)$$

where η is the bargaining power of the firm and all the functions have been defined above.

In solving this problem, the firm takes as given the optimal policy of new firms determining the repudiation values and the first order condition with

respect to h_1 reads:

$$(1 - \alpha) \left(\frac{k_1}{h_1} \right)^\alpha = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0} \right), \quad (10)$$

where the subscripts denote derivatives.

The optimal policy of a new firm, instead, is determined by the first order condition to problem (5), that is:

$$(1 - \alpha) \left(\frac{k_1}{h_1} \right)^\alpha = \varphi_{h_1}(h_0, h_1) + \tau. \quad (11)$$

The left-hand terms in (10) and (11) are the marginal productivity of knowledge. The right-hand terms are the marginal costs. For an incumbent firm, the marginal cost derives from the effort incurred by the worker plus the obsolescence of physical capital. For a new firm the obsolescence cost is replaced by the entry cost. These two conditions clearly show the different incentives to invest for an incumbent versus a new firm. On the one hand, new firms do not have any physical capital and knowledge upgrading does not generate capital obsolescence. On the other, they must pay the entry cost τh_1 .

It is important to point out that the bargaining powers of firms and workers do not matter for the choice of the investment in knowledge. This only matters for the division of the surplus. In case of new firms the whole surplus goes to the worker. For incumbent firms it is split according to the bargaining power η .

Let h_1^{Old} be the optimal knowledge investment of an incumbent old firm (the solution to condition (10)) and h_1^{New} the optimal investment of a new firm (the solution to condition (11)). The following proposition formalizes the relation between barriers to entry and knowledge investment.

Proposition 1 *The knowledge investment of a new firm, h_1^{New} , is strictly decreasing in the entry cost τ and there exists $\bar{\tau} > 0$ such that $h_1^{New} = h_1^{Old}$.*

Proof 1 *The first order condition for the choice of k_1 is $\alpha(k_1/h_1)^{\alpha-1} = 1$ for both incumbent and new firms. Using this condition, (10) and (11) become:*

$$\begin{aligned} (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_{h_1}(h_0, h_1^{Old}) + \delta \cdot \left(\frac{k_0}{h_0} \right) \\ (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_{h_1}(h_0, h_1^{New}) + \tau. \end{aligned}$$

The proposition follows directly from these two conditions.

Q.E.D.

In equilibrium there is no entrance of new firms and the investment in knowledge is $h_1 = h_1^{Old}$. The potential entrance of new firms only affects the payment received by the worker. Taking the first order conditions with respect to w in problem (9) and rearranging we get:

$$w = (1 - \eta) \cdot \left\{ \left[1 - \delta \cdot \left(\frac{h_1}{h_0} \right) \right] k_0 + h_1^{1-\alpha} k_1^\alpha - k_1 - D_f(k_0, h_0) \right\} + \eta \cdot \left\{ \varphi(h_0, h_1) + D_w(h_0) \right\}. \quad (12)$$

Therefore, when the investor commits to the contract, the potential entrance of new firms does not affect the investment in knowledge. This also implies that the entry barriers τ are irrelevant for the equilibrium investment.

This result also holds when both parties commit to the contract, that is, they do not renegotiate after the investment. Because h_1^{Old} maximizes the total surplus, this must also be the equilibrium investment if both parties commit to the contract. The same result applies if it is the worker who commits. In this case the investor can renege on the promised payments after the investment in knowledge. However, this problem can be solved by making the payment w before the investment. As long as the contract is enforceable for the worker, there is no risk that she runs away or does not exercise the effort to acquire knowledge after receiving w .

4.3 Equilibrium with double-sided limited commitment

We want to show first that, when the investor can not commit to fulfill his promises, he will renegotiate the contract after the choice of h_1 . To see this, we must derive the value that the worker would get by quitting the firm when her knowledge has already been chosen to be h_1 . This is the surplus generated by a new firm that hires a worker as defined in (8).

Renegotiation takes the form of bargaining over the terms of the contract. The bargaining solution solves the problem:

$$\max_{\hat{w}} \left[\widehat{W}(h_1, k_1) - \widehat{D}_w(h_1) \right]^{1-\eta} \left[\widehat{J}(h_1, k_1) - \widehat{D}_f(h_1, k_1) \right]^\eta \quad (13)$$

where the surpluses in the second stage are defined in (2) and (4).

Taking first order conditions with respect to \hat{w} and solving, the wage received by the worker is:

$$\hat{w} = (1 - \eta) \left[f(h_1, k_1) - \widehat{D}_f(k_1, h_1) \right] + \eta \widehat{D}_w(h_1) \quad (14)$$

We can now use the definitions of the repudiation values provided in (6) and (8) to eliminate $\widehat{D}_w(h_1)$ and $\widehat{D}_f(k_1, h_1)$ and express the wage received by the worker after renegotiation as:

$$\hat{w} = f(h_1^{Old}, k_1^{Old}) - k_1^{Old} - \tau h_1^{Old} \quad (15)$$

Therefore, if the worker decides to stay with the current employer at the beginning of the period, she will receive the utility:

$$\hat{w} - \varphi(h_0, h_1^{Old}) = f(h_1^{Old}, k_1^{Old}) - k_1^{Old} - \tau h_1^{Old} - \varphi(h_0, h_1^{Old}) \quad (16)$$

If instead the worker chooses to quit for a new firm at the beginning of the period she will get:

$$\max_{h_1, k_1} \left\{ f(h_1, k_1) - k_1 - \tau h_1 - \varphi(h_0, h_1) \right\} = f(h_1^{New}, k_1^{New}) - k_1^{New} - \tau h_1^{New} - \varphi(h_0, h_1^{New}) \quad (17)$$

As long as $h_1^{Old} \neq h_1^{New}$ (and $k_1^{Old} \neq k_1^{New}$), the value of quitting at the beginning of the period is higher than the value of staying with the incumbent firm and agreeing to the policy with commitment. The only way to retain the worker is for the incumbent firm to agree to the same knowledge investment chosen by a new firm; that is, h_1^{New} . In this way the worker keeps the repudiation value high and prevents the firm from renegotiating.

Proposition 2 *Suppose that all firms have the same initial states (k_0, h_0) . Then there is a unique equilibrium with aggregate knowledge $H_1 = h_1^{New}$.*

Proof 2 *It is enough to show that with any $h_1 \neq h_1^{New}$, the worker will get a lower utility. This must be the case because the value received by the worker if she stays with the current employer, defined in (16), is maximized at h_1^{New} .*

Because h_1^{New} is decreasing in τ (as established in Proposition 1), the accumulation of knowledge decreases with the cost of entry. Therefore, with double-sided limited enforcement, there is a negative correlation between barriers to entry and the accumulation of knowledge. This is in contrast to the equilibrium with commitment of at least one side. In this case the accumulation of knowledge is $H_1 = h_1^{Old}$. As we have seen in the previous

section, h_1^{Old} is independent of τ , and therefore, the equilibrium does not depend on barriers to entry.

To summarize, when contracts are not enforceable for both workers and investors, greater competition (lower barriers to entry) leads to higher investment in knowledge. Because the investment is determined by the optimality condition of new firms, this level is not necessarily efficient for incumbent firms. In particular, if τ is small, incumbent firms accumulate too much knowledge. The presence of spillovers, however, may make the higher investment socially desirable as we will see in the analysis of the general model.

5 The infinite horizon model

In this section we generalize the model to an infinite horizon set-up. There are two important gains from extending the analysis to the infinite horizon. First, it allows us to derive the initial conditions k_0 and h_0 endogenously as steady state values. Second, an infinite horizon structure is better suited for the quantitative analysis of Section 6.

We first characterize the equilibrium with commitment and then we turn to the case of double-sided limited commitment. The comparison between these two environments clarifies the importance of double-sided limited enforcement for the entry barriers to affect the accumulation of knowledge. To present the results more compactly, we relegate most of the technicalities and proofs to the appendix.

Before continuing, it will be convenient to define the gross output function, inclusive of undepreciated capital, as follows:

$$\pi(h_t, k_t, h_{t+1}) = h_t^{1-\alpha} k_t^\alpha + \left[1 - \delta \cdot \left(\frac{h_{t+1}}{h_t} \right) \right] k_t. \quad (18)$$

5.1 Some preliminaries

The analysis with the infinite horizon will concentrate on steady state equilibria. Therefore, we will ignore the aggregate states as an explicit argument of the value functions.

Although in equilibrium there is no entrance of firms, we still need to solve for the dynamics of a new firm in order to determine the outside or repudiation values for workers and firms. Even if the analysis is limited to steady states, newly created firms do experience a transition to the long-term level of physical and knowledge capital.

Let's start with the definition of the surpluses for workers and firms. These are defined as:

$$W(h_t, k_t) = \sum_{j=t}^{\infty} \beta^{j-t} [w_j - \varphi(h_j, h_{j+1}; H)] \quad (19)$$

$$\widehat{W}(h_{t+1}, k_{t+1}) = w_t + \sum_{j=t+1}^{\infty} \beta^{j-t} [w_j - \varphi(h_j, h_{j+1}; H)] \quad (20)$$

$$J(h_t, k_t) = \sum_{j=t}^{\infty} \beta^{j-t} [\pi(h_j, k_j, h_{j+1}; H) - w_j - k_{j+1}] \quad (21)$$

$$\widehat{J}(h_{t+1}, k_{t+1}) = -w_t + \sum_{j=t+1}^{\infty} \beta^{j-t} [\pi(h_j, k_j, h_{j+1}; H) - w_j] \quad (22)$$

Keeping the assumption of competition among new entrants, the repudiation values of workers are the surpluses generated by new firms. At the beginning of period t , the surplus is given by:

$$D_w(h_t) = \max_{\{k_{j+1}, h_{j+1}\}_{j=t}^{\infty}} \left\{ -\tau h_{t+1} - k_{t+1} - \varphi(h_t, h_{t+1}; H) + \sum_{j=t+1}^{\infty} \beta^{j-t} [\pi(h_j, k_j, h_{j+1}) - k_{j+1} - \varphi(h_j, h_{j+1}; H)] \right\} \quad (23)$$

After the investment in knowledge, the repudiation value is:

$$\widehat{D}_w(h_{t+1}) = \max_{k_{t+1}, \{k_{j+1}, h_{j+1}\}_{j=t+1}^{\infty}} \left\{ -\tau h_{t+1} - k_{t+1} + \sum_{j=t+1}^{\infty} \beta^{j-t} [\pi(h_j, k_j, h_{j+1}) - k_{j+1} - \varphi(h_j, h_{j+1}; H)] \right\} \quad (24)$$

Again, this is the surplus generated by a new firm that hires a worker with knowledge capital h_{t+1} .

The key difference between a firm entering at the beginning of the period and after the investment in knowledge is that the effort to accumulate knowledge has already been exercised and h_{t+1} is given at this point. This explains why the choice of knowledge starts in the next period.

The repudiation values of firms are given by:

$$D_f(h_t, k_t) = \pi(h_t, k_t, h_{t+1}^{New}) + \tau h_{t+1}^{New} \quad (25)$$

$$\widehat{D}_f(h_{t+1}, k_{t+1}) = k_{t+1} + \tau h_{t+1} \quad (26)$$

where h_{t+1}^{New} is the initial investment in knowledge chosen by a new firm created at the beginning of period t .

Using these functions we can now look at the special cases of limited commitment from one or both sides.

5.2 Equilibrium with one-sided commitment

As in the one-period model, the equilibrium allocation with investor's commitment is equivalent to the allocation achieved when the worker commits (with or without commitment from the investor). We will concentrate on the case with investor's commitment.

The optimal solution for a new first is characterized by the first order conditions to problem (23). Because of the entry cost and the obsolescence of physical capital, the optimality conditions in the entry period differ from the optimality conditions in subsequent periods. The first order conditions at t when the firm is started are:

$$\beta \pi_2(h_{t+1}, k_{t+1}, h_{t+2}) = 1 \quad (27)$$

$$\tau + \varphi_2(h_t, h_{t+1}; H) = \beta \left[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \right], \quad (28)$$

where subscripts denote derivatives. The first condition equalizes the gross marginal return of capital to its marginal cost, which is 1. The last condition equalizes the marginal cost of accumulating knowledge to the discounted value of its return (greater production and lower cost of future knowledge investment).

The first order conditions after entering are:

$$\beta \left[\pi_2(h_{t+1}, k_{t+1}, h_{t+2}) \right] = 1 \quad (29)$$

$$-\pi_3(h_t, k_t, h_{t+1}) + \varphi_2(h_t, h_{t+1}; H) = \beta \left[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \right]. \quad (30)$$

along a transversality condition.

Conditions (28) and (30) show the asymmetry between new and incumbent firms. While the marginal benefit from investing in knowledge (the right-hand side) is the same, the marginal cost (the left-hand side) differs. For new firms this includes the entry cost, τ . For incumbent firms the entry cost is replaced by the depreciation of physical capital, $-\pi_3(h_t, k_t, h_{t+1})$.

We can now characterize the steady state. Because in equilibrium there is no entrance, all firms will have the economy-wide knowledge H . The convergence to the economy-wide average is the result of the spillovers in the accumulation of knowledge. Because of this, firms with lower than average knowledge tend to invest more. Thanks to the complementarity of knowledge and physical capital, all firms also accumulate the same economy-wide level of physical capital K . The values of H and K are determined by (27) and (30) after imposing the steady state conditions, that is:

$$\beta\pi_2(H, K, H) = 1 \quad (31)$$

$$-\pi_3(H, K, H) + \varphi_2(H, H; H) = \beta \left[\pi_1(H, K, H) - \varphi_1(H, H; H) \right] \quad (32)$$

Appendix B shows that the steady state values of H and K are unique.

After solving for H and K , we can then solve for the steady state payment w . This requires us to solve for the whole transition experienced by a 'new firm', as characterized by the first order conditions (27)-(30). Even if in equilibrium workers do not quit and new firms are not created, the payment w must satisfy the enforcement constraints which depend on the surplus created by new firms, that is, $W(H) \geq D_w(h_t)$ and $\widehat{W}(h_{t+1}, k_{t+1}) \geq \widehat{D}_w(h_{t+1}, k_{t+1})$.

Conditions (31) and (32) also reveal that the entry cost τ does not affect the steady state values of K and H . We will see in the next section that this does not hold in the case of double-sided limited commitment.

5.3 Equilibrium with double-sided limited commitment

With double-side limited commitment, the contract is renegotiated in every period, before and after the investment in knowledge. Renegotiation involves bargaining over the respective surpluses as defined in Section 5.1. Because bargaining takes place in every period, it will be convenient to rewrite the surpluses recursively:

$$W(h_t, k_t) = w_t - \varphi(h_t, h_{t+1}; H) + \beta W(h_{t+1}, k_{t+1}) \quad (33)$$

$$\widehat{W}(h_{t+1}, k_{t+1}) = \hat{w}_t + \beta W(h_{t+1}, k_{t+1}) \quad (34)$$

$$J(h_t, k_t) = \pi(h_t, k_t, h_{t+1}; H) - w_t - k_{t+1} + \beta J(h_{t+1}, k_{t+1}) \quad (35)$$

$$\widehat{J}(h_{t+1}, k_{t+1}) = -\hat{w}_t + \beta J(h_{t+1}, k_{t+1}) \quad (36)$$

where the optimal policies are determined through bargaining.

Using these definition of the surpluses, the repudiation values for the worker can be written as:

$$D_w(h_t) = \max_{h_{t+1}, k_{t+1}} \left\{ -\tau h_{t+1} - k_{t+1} - \varphi(h_t, h_{t+1}; H) + \beta W(h_{t+1}, k_{t+1}) + \beta J(h_{t+1}, k_{t+1}) \right\} \quad (37)$$

$$\widehat{D}_w(h_{t+1}) = \max_{k_{t+1}} \left\{ -\tau h_{t+1} - k_{t+1} + \beta W(h_{t+1}, k_{t+1}) + \beta J(h_{t+1}, k_{t+1}) \right\} \quad (38)$$

If the contract is renegotiate only at the beginning of the period, the bargaining problem solves:

$$\max_{w_t, h_{t+1}, k_{t+1}} \left[W(h_t, k_t) - D_w(h_t) \right]^{1-\eta} \left[J(h_t, k_t) - D_f(h_t, k_t) \right]^\eta \quad (39)$$

Let's denote by w_t^{Old} , h_{t+1}^{Old} and k_{t+1}^{Old} the solutions of this bargaining problem. However, if the contract is renegotiated after the investment, the actual wage will be determined by the bargaining problem

$$\max_{\hat{w}_t} \left[\widehat{W}(h_{t+1}, k_{t+1}) - \widehat{D}_w(h_{t+1}) \right]^{1-\eta} \left[\widehat{J}(h_{t+1}, k_{t+1}) - \widehat{D}_f(h_{t+1}, k_{t+1}) \right]^\eta, \quad (40)$$

which takes as given the new levels of human and physical capital.

What we want to show is that, if the contract is renegotiated after the investment in knowledge, the worker will receive less utility than the utility she would get by quitting the firm at the beginning of the period. This is stated in the following proposition.

Proposition 3 *If the contract is renegotiated after the investment in knowledge, a worker that stays with the firm and agrees to the investment h_{t+1}^{Old} will receive less utility than a worker that quits the firm at the beginning of the period.*

Proof 3 See Appendix C.

A direct implication of this proposition is stated in the following lemma.

Lemma 1 *With double-sided limited commitment, the knowledge investment chosen by an incumbent firm is equal to the knowledge investment chosen by a new firm; that is, $h_{t+1}^{Old} = h_{t+1}^{New} = f(h_t)$.*

Proof 1 *It follows directly from Proposition 3.*

Since the firm can renegotiate the promised payments after the investment in knowledge and in doing so the worker would receive less utility, the worker would not stay unless the firm agrees to the same knowledge investment chosen by a new firm. In this way, the worker keeps the outside value high and prevents the firm from renegotiating.

Let $V(h_t)$ be the repudiation value for the worker when neither the investor nor the worker commit to the contract. Given the above proposition, the optimization problem for a new firm started at t can be written as:

$$V(h_t) = \max_{h_{t+1}, \{k_{j+1}\}_{j=t}^{\infty}} \left\{ -\tau h_{t+1} - k_{t+1} - \varphi(h_j, h_{j+1}; H) + \sum_{j=t+1}^{\infty} \beta^{t-j} \left[\pi(h_j, k_j, h_{j+1}) - k_{j+1} - \varphi(h_j, h_{j+1}; H) \right] \right\} \quad (41)$$

subject to

$$h_{j+1} = f(h_j), \quad \text{for } j > t. \quad (42)$$

Notice that only the initial knowledge h_{t+1} is chosen in this problem. Future values are determined by the investment policy of future new firms; that is, $h_{j+1} = f(h_j)$, for $j > t$.

Conditions (27) and (29), derived in the environment with investor's commitment, are also valid in the case with double-sided limited commitment.

The optimality condition for the accumulation of knowledge, however, is different. For new firms this is given by:

$$\begin{aligned} \tau + \varphi_2(h_t, h_{t+1}; H) = & \beta \left\{ \pi_1(h_{t+1}, k_{t+1}, f(h_{t+1})) - \varphi_1(h_{t+1}, f(h_{t+1}); H) \right. \\ & \left. + \left[\pi_3(h_{t+1}, k_{t+1}, f(h_{t+1})) + \tau \right] f_1(h_{t+1}) \right\} \end{aligned} \quad (43)$$

For incumbent firms there is no optimality condition for the investment in knowledge since they take as given the investment policy $f(h_t)$.

Imposing the steady state conditions $h_t = h_{t+1} = H$ and $k_t = k_{t+1} = K$, conditions (27) and (43) become:

$$\beta \pi_2(H, K, H) = 1 \quad (44)$$

$$\begin{aligned} \tau + \varphi_2(H, H; H) = & \beta \left\{ \pi_1(H, K, f(H)) - \varphi_1(H, f(H); H) \right. \\ & \left. + f_1(H) \left[\pi_3(H, K, f(H)) + \tau \right] \right\}. \end{aligned} \quad (45)$$

Unlike the case in which the investor commits to the contract, these two conditions are no longer sufficient to determine the steady state values of H and K . The unknown function $f(H)$ also needs to be determined. This requires us to solve for a fixed point problem. Denote by $h' = \psi(h; f)$ the policy function that solves problem (41), for given f . The policy function satisfies the first order condition (43) and in equilibrium $f(H) = \psi(H; f)$.

Because incumbent firms innovate at the same rate as new firms, condition (43) also determines the investment in knowledge of incumbent firms. Therefore, in order to determine whether the lack of commitment leads to higher or lower investment in knowledge, we have to compare condition (45) to the optimality condition when the investor commits to the long-term contract, that is, condition (30).

Let H^C be the steady state knowledge in the economy in which the investor commits, and H^{NC} the steady state knowledge without commitment. We then have the following proposition:

Proposition 4 *Suppose that $f_1(H) \leq 1$. Then the steady state value of H^{NC} is strictly decreasing in τ and there exists $\bar{\tau} > 0$ such that $H^{NC} > H^C$ for $\tau < \bar{\tau}$ and $H^{NC} < H^C$ for $\tau > \bar{\tau}$.*

Proof 4 See Appendix D.

Notice that the proof is based on the assumption that $f_1(H) \leq 1$; that is, the derivative of the policy function at the steady state equilibrium is not greater than one. We have checked this condition numerically.

To summarize, when contracts are not enforceable neither for the worker nor for the investor, barriers to entry are harmful for the accumulation of knowledge. With low barriers, the economy experiences a higher level of income than in the case with commitment. This could be welfare-improving if there are spillovers in the accumulation of knowledge.

6 Quantitative application

In this section we use the model to quantify the contribution of the cost of business start-up in generating cross-country income inequality. In the quantitative application we focus on the ‘cost of business start-up’ because of data availability. It should be clear, however, that our theory applies more broadly to other barriers affecting the mobility of knowledge.

We calibrate the economy to the United States and then we ask how much of the cross-country income gap from the US can be accounted for by the observed cost of business start-up. The discount factor, β , the production parameter, α , and the depreciation parameter, δ , are calibrated to replicate the following moments: an interest rate of 5 percent, a capital income share of 33 percent, and a capital-output ratio of 3. This implies $\beta = 0.9524$, $\alpha = 0.33$, and $\delta = 0.06$. Notice that the three moments are invariant to the entry barrier τ , and therefore, they are constant across countries.⁶

The effort cost function is derived from the accumulation equation for the stock of knowledge, which is assumed to take the form:

$$h_{t+1} = (1 - \phi)h_t + (H_t^\theta e_t^{1-\theta})^\nu,$$

⁶While it is easy to see the mapping between the first two moments and the first two parameters ($\beta = 1/(1+r)$ and $\alpha = rK/Y$), less obvious is the mapping between δ and the capital-income ratio. From condition(27), evaluated at the steady state, we have $\beta\pi_2(H, K, H) = \beta[1 - \delta + \alpha(K/H)^{\alpha-1}] = 1$. Given the output function $Y = H^{1-\alpha}K^\alpha$, the capital-output ratio can be written as $K/Y = (K/H)^{1-\alpha}$. Using this expression to eliminate K/H in the previous condition, we get $\beta[1 - \delta + \alpha/(K/Y)] = 1$. Therefore, after choosing β and α , the parameter δ is uniquely determined by the capital-output ratio.

where H_t is the average level of knowledge, e_t is the effort cost of accumulating knowledge and ϕ is the depreciation rate. The parameter $\nu < 1$ captures the return to scale in the accumulation of knowledge and $\theta < 1$ the leakage or spillover effects. Inverting, we get the cost function:

$$e_t = \varphi(h_t, h_{t+1}; H_t) = \frac{[h_{t+1} - (1 - \phi)h_t]^{\frac{1}{(1-\theta)\nu}}}{H_t^{\frac{\theta}{1-\theta}}},$$

which is homogeneous of degree $\rho = (1 - \theta\nu)/(1 - \theta)\nu$.

The depreciation of knowledge results from working directly with the stationary version of the model, detrended by the rate of worldwide knowledge. The parameter ϕ is then approximately equal to the exogenous rate of growth.⁷ Assuming that the economy grows at 1.8 percent per year, we set $\phi = 0.018$. This is about the average growth rate in per-capita GDP experienced by the US during the last century.

The values of the other two parameters, θ and ν , are more controversial. Manuelli & Seshadri (2005) uses a similar specification of the investment function, within an overlapping generation model, but without externalities. In order to generate some key properties of the life-time profile of earnings, they choose a return to scale of 0.93. This is also the value estimated by Heckman, Lochner, & Taber (1998). We use this value to calibrate ν on the assumption that there is sufficient intergenerational transmission of human capital.⁸ For the baseline parametrization we also follow Manuelli & Seshadri (2005) and assume no externalities, that is, $\theta = 0$. The sensitivity analysis will clarify how the results depend on the choice of θ and ν .

⁷The original (undetrended) function for the accumulation of knowledge is $h_{t+1} = h_t + \bar{H}_t^{1-\nu}(H_t^\theta e_t^{1-\theta})^\nu$, where \bar{H}_t is the worldwide knowledge, external to an individual country, which grows at the constant rate \bar{g} . Normalizing all terms by \bar{H}_t , the investment function becomes $h_{t+1} = (1 - \phi)h_t + A(H_t^\theta e_t^{1-\theta})^\nu$, where $\phi = \bar{g}/(1 + \bar{g}) \simeq \bar{g}$ and $A = 1/(1 + \bar{g})$. Because A acts as a rescaling factor, we can set $A = 1$.

⁸In Manuelli & Seshadri (2005) the cost of human capital investment has two components: time and expenditures. Our specification does not distinguish between these components and the investment cost is captured by the single variable e . However, this does not alter in important ways the main properties of the model. As Manuelli and Seshadri show, the key parameter to replicate the life-time earning profile is not the relative importance of the two inputs but the return-to-scale parameter. Notice also that the depreciation rate $\phi = 0.018$ is also equal to the value chosen by Manuelli and Seshadri.

start-up has no impact on the equilibrium income; that is, $\hat{y}_i = y_{US}$. For the baseline calibration the index is 0.51. Therefore, the model accounts for roughly half of the cross-country income gaps from the US.

Next we show how the values of θ and ν affect the results. Table 2 reports the income gaps accounted for by the model for alternative values of these parameters. The general finding is that the model is more successful the higher the return to scale, ν , and the lower the externalities, θ . The sensitivity is especially high for the return to scale. However, even for small returns to scale, the model accounts for a non-negligible fraction of cross-country income gaps. Even if we take the extreme parametrization chosen by Parente & Prescott (2002), $\nu = 0.6$, the model still accounts for about 11 percent of the income gaps.

Table 2: Income gaps accounted for by the model.

Value of θ	Value of ν					
	<i>0.97</i>	<i>0.93</i>	<i>0.90</i>	<i>0.80</i>	<i>0.70</i>	<i>0.60</i>
<i>0.0</i>	0.68	0.51	0.42	0.26	0.18	0.13
<i>0.1</i>	0.66	0.48	0.40	0.25	0.17	0.12
<i>0.2</i>	0.64	0.46	0.37	0.23	0.15	0.11
<i>0.3</i>	0.62	0.43	0.35	0.21	0.14	0.10
<i>0.4</i>	0.59	0.40	0.32	0.19	0.12	0.09

We have also calculated the ‘domestic socially optimal’ steady-state level of output, that is, the output resulting from solving the problem of a country’s planner. This differs from the competitive output because of the externality at the domestic level, represented by θ . It also differs from the ‘global socially optimal’ steady-state level of output, which is the solution to the problem of a ‘global planner’, which internalizes the worldwide leakage, or spillover, represented by ν . The steady-state values of H and K in the domestic planner allocation are found by solving the first order conditions:

$$\beta\pi_2(H, K, H) = 1$$

$$\varphi_2(H, H; H) - \pi_3(H, K, H) = \beta \left[\pi_1(H, K, H) - \varphi_1(H, H; H) - \varphi_3(H, H; H) \right].$$

These are similar to conditions (31) and (32) except for the additional term $\varphi_3(H, H; H)$ in the second equation. This term captures the externality taken into account by the planner but ignored by the atomistic agents.

Table 3 reports the ‘competitive’ output as a fraction of the ‘domestic socially optimal’ output when there are no barriers to entry. A value greater than 1 means that there is over-accumulation of knowledge compared to the socially optimal level. As expected, this arises when the spillovers are small or zero; that is, when θ is small. In this case moderate barriers to business start-up would be welfare improving. On the other hand, values smaller than 1 mean that there is under-accumulation of knowledge compared to the socially optimal level. In this case barriers to entry are always suboptimal, while moderate subsidies could improve welfare. As can be seen from the table, the under-accumulation of knowledge arises for moderate spillovers.

Table 3: Steady-state output when contracts are not enforceable and there are no barriers to entry. Numbers are relative to the socially optimal output.

Value of θ	Value of ν					
	<i>0.97</i>	<i>0.93</i>	<i>0.90</i>	<i>0.80</i>	<i>0.70</i>	<i>0.60</i>
<i>0.0</i>	1.81	1.28	1.18	1.08	1.04	1.03
<i>0.1</i>	0.80	0.92	0.95	0.99	1.00	1.00
<i>0.2</i>	0.41	0.71	0.80	0.92	0.96	0.98
<i>0.3</i>	0.25	0.58	0.70	0.88	0.94	0.97
<i>0.4</i>	0.17	0.51	0.64	0.84	0.92	0.96

7 Covenants and other barriers to mobility

Other barriers to the mobility of knowledge capital may have an effect in our model which is similar to the cost of business start-up. As we have discussed in the Introduction, even within a similar legal and economic environment—resulting in similar costs for business start-up—there may be differences in other barriers. Covenants is one of them. A covenant which is ex-post enforced prevents the worker from using her acquired knowledge if she moves to another firm.

A natural way to model non-competitive covenants is by assuming that a quitting worker can only use a fraction ξ of her accumulated knowledge in a new firm. This formulation also captures the case in which part of the knowledge can not be used by the worker due to the enforcement of IPR if she does not have full control of the patent. In our formulation, a more stringent enforcement of covenants (or IPRs) is captured by a lower fraction ξ .

To keep the presentation brief, we limit the analysis to the one-period model. The extension to an infinite horizon will follow the same logic as in the analysis with entry costs. The problem solved by a new firm which started at the beginning of the period can be written as:

$$D_w(h_0) = \max_{h_1, k_1} \left\{ -\varphi(h_0, h_1) - k_1 + (\xi h_1)^{1-\alpha} k_1^\alpha \right\} \quad (46)$$

The problem solved by an incumbent firm is as in problem (9). The first order conditions with respect to h_1 , for incumbent and new firms respectively, are:

$$(1 - \alpha) \left(\frac{k_1}{h_1} \right)^\alpha = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0} \right) \quad (47)$$

$$(1 - \alpha) \left(\frac{k_1}{h_1} \right)^\alpha = \varphi_{h_1}(h_0, h_1) \cdot \xi^{\alpha-1}. \quad (48)$$

Because $\xi < 1$ and $\alpha < 1$, the term $\xi^{\alpha-1} > 1$. Therefore, covenants have the effect of increasing the cost of accumulating knowledge and act similarly to the entry cost τ . Proposition 1 becomes:

Proposition 5 *The knowledge investment of a new firm h^{New} is strictly increasing in ξ and there exists $\bar{\xi} > 0$ such that $h^{New} = h^{Old}$.*

Proof 5 Using the first order condition for the choice of physical capital, which is $\alpha(k_1/h_1)^{\alpha-1} = 1$ for both incumbent and new firms, the above first order conditions can be rewritten as:

$$\begin{aligned} (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_{h_1}(h_0, h^{Old}) + \delta \cdot \left(\frac{k_0}{h_0}\right) \\ (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_{h_1}(h_0, h^{New})\xi^{-1} \end{aligned}$$

The proposition follows directly from these two conditions.

Q.E.D.

All the results obtained in Section 4 trivially extend to the case of covenants and other similar barriers to the mobility of knowledge.

8 Conclusion

We have developed a theory in which *barriers to knowledge mobility* affect the accumulation of knowledge, and therefore the level of income. The theory does not simply say that “competition enhances income”. It shows how different forms of contract enforcement affect the relation between competition, accumulation of human capital and economic development. In particular, when both investors and workers can not commit to long-term contracts, the accumulation of human capital is determined by those firms that value human capital the most, that is, start-up firms. As a result, high levels of human capital accumulation are associated with low barriers to the mobility of knowledge. In the absence of barriers the accumulation of knowledge can be suboptimal at the firm level but could be welfare improving if there are spillovers.

In a semi-endogenous growth model we have shown that *barriers to business start-up* have the potential to explain significant cross-country income differences. This is the first step to bringing our theory to the data. We have also shown that other *barriers to knowledge mobility*, such as strict enforcement of Covenants or Intellectual Property Rights, can have similar effects, suggesting a wide scope for the empirical application of the theory.

A Bargaining over the surplus of new firms

We generalize the results of the one period model to the case in which the surplus generated by new firms is also bargained between the worker and the firm. This can be justified by assuming that the determination of wages takes place after the worker matches with the investor.

Suppose that the number of matches are determined by $\min\{v, u\}$ where v is the number of vacancies and u the number of searchers. Also assume that posting a vacancy requires a cost κ . These are the typical assumptions in standard matching models of the labor market. The only particularity is the Leontif structure of the matching function. With this functional form the probability that a searching worker finds a job is always 1 and in equilibrium there is no unemployment. This is without loss of generality for the analytical results of the paper.

Denote by ζ the bargaining power of new firms. The baseline model analyzed in the main body of the paper is obtained by setting $\zeta = 0$. Another special case is when $\zeta = \eta$, that is, the bargaining power is the same for new and incumbent firms.

When the investor commits not to renegotiate the contract after the investment in knowledge, the optimal policy solves the bargaining problem,

$$\max_{w, h_1, k_1} \left[W(h_0, k_0) - D_w(h_0) \right]^{1-\eta} \left[J(h_0, k_0) - D_f(h_0, k_0) \right]^\eta \quad (49)$$

What changes is the determination of the outside values for workers. This is still determined by the value obtained from a new firm. Now, however, the worker does not get the whole surplus. More specifically, the new firm solves the bargaining problem:

$$\max_{w, h_1, k_1} \left[w - \varphi(h_0, h_1) \right]^{1-\zeta} \left[-w - k_1 - \tau h_1 + f(h_1, k_1) \right]^\zeta \quad (50)$$

After the investment in knowledge the repudiation value of the worker is $\widehat{D}_w(h_1) = \widehat{w}^{New}$ where \widehat{w}^{New} is determined by the bargaining problem:

$$\max_{w, k_1} w^{1-\zeta} \left[-w - k_1 - \tau h_1 + f(h_1, k_1) \right]^\zeta \quad (51)$$

The repudiation values can also be written as:

$$D_w(h_0) = (1 - \zeta) \max_{h_1, k_1} \left\{ -\varphi(h_0, h_1) - \tau h_1 - k_1 + h_1^{1-\alpha} k_1^\alpha \right\} \quad (52)$$

$$\widehat{D}_w(h_1) = (1 - \zeta) \max_{k_1} \left\{ -\tau h_1 - k_1 + h_1^{1-\alpha} k_1^\alpha \right\} \quad (53)$$

that is, they are the worker's share of the total net surpluses created by a new firm before and after the investment in knowledge and the investment is chosen to maximize the surplus. The repudiation values for firms is as in (7) and (8).

The optimal investment policy with commitment of an incumbent firm is characterized by the first order conditions with respect to h_1 in problem (49). The firm takes as given the optimal policy of new firms determining the repudiation values and the first order conditions read:

$$(1 - \alpha) \left(\frac{k_1}{h_1} \right)^\alpha = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0} \right). \quad (54)$$

The optimal policy of a new firm, instead, is determined by the first order condition to problem (50), that is:

$$(1 - \alpha) \left(\frac{k_1}{h_1} \right)^\alpha = \varphi_{h_1}(h_0, h_1) + \tau. \quad (55)$$

These are the same conditions (10) and (11) obtained before. Consequently, Proposition (1) also applies here. The wage paid to the worker is given by (12).

A.1 Double-side limited commitment

We turn now to the case of double-side limited commitment. Let's show first that, in absence of commitment, the investor will renegotiate the contract after the choice of h_1 . Renegotiation takes the form of bargaining over the terms of the contract. The bargaining solution solves the problem:

$$\max_{\hat{w}} \left[\widehat{W}(h_1, k_1) - \widehat{D}_w(h_1) \right]^{1-\eta} \left[\widehat{J}(h_1, k_1) - \widehat{D}_f(h_1, k_1) \right]^\eta \quad (56)$$

where the surpluses in the second stage are defined in (2) and (4). Notice that at this stage the bargaining is only over the payment of wages because the investment in knowledge and physical capital has already been chosen.

Taking first order conditions with respect to \hat{w} and solving we get:

$$\hat{w} = (1 - \eta) \left[f(h_1, k_1) - \widehat{D}_f(k_1, h_1) \right] + \eta D_w(h_1) \quad (57)$$

Using the definition of $\widehat{D}_w(h_1)$ and $\widehat{D}_f(k_1, h_1)$ and noticing that in (53) the solution for physical capital is k_1^{Old} when $h_1 = h_1^{Old}$, the wage received by the worker can be written as:

$$\hat{w} = (1 - \zeta\eta) \left[f(h_1^{Old}, k_1^{Old}) - k_1^{Old} - \tau h_1^{Old} \right] \quad (58)$$

Therefore, if the worker decides to stay with the current employer at the beginning of the period, she will receive the utility:

$$\hat{w} - \varphi(h_0, h_1^{Old}) = (1 - \zeta\eta) \left[f(h_1^{Old}, k_1^{Old}) - k_1^{Old} - \tau h_1^{Old} \right] - \varphi(h_0, h_1^{Old})$$

This needs to be compared with the utility that the worker will receive if she quits for a new firm at the beginning of the period. Also a new firm can renegotiate the contract in the second stage. Therefore, the worker that matches with a new firm at the beginning of the period and agrees to the policy h_1 and k_1 , anticipates that the utility she will receive (given the renegotiation) is:

$$\hat{w} - \varphi(h_0, h_1) = (1 - \zeta\eta) \left[f(h_1, k_1) - k_1 - \tau h_1 \right] - \varphi(h_0, h_1)$$

Obviously, the worker will agree to stay in the new match only if the investment maximizes this utility. If the firm does not agree to this investment, the worker will choose the optimal h_1 and re-match with a new firm in the second stage. Hence, the utility obtained by quitting at the beginning of the period is:

$$\begin{aligned} \max_{h_1, k_1} \left\{ (1 - \zeta\eta) \left[f(h_1, k_1) - k_1 - \tau h_1 \right] - \varphi(h_0, h_1) \right\} = \\ (1 - \zeta\eta) \left[f(h_1^{New}, k_1^{New}) - k_1^{New} - \tau h_1^{New} \right] - \varphi(h_0, h_1^{New}), \end{aligned}$$

where h_1^{New} is the knowledge investment that maximizes the utility of the worker.

As long as $h_1^{Old} \neq h_1^{New}$ (and $k_1^{Old} \neq k_1^{New}$), the value of quitting at the beginning of the period is higher than the value of staying with the incumbent firm and agreeing to the policy with commitment. The only way to retain the worker is for the incumbent firm to agree to the same knowledge investment chosen by a new firm; that is, h_1^{New} . In this way the worker keeps the repudiation value high and prevents the firm from renegotiating. Therefore, Proposition 2 still applies even if the surplus of a new firm is shared according to the bargaining share ζ .

B Steady state equilibrium when the investor commits

Proposition 6 *There is a unique steady state equilibrium in which all firms have the same knowledge H and physical capital K .*

Proof 6 *Consider condition (32), which we rewrite here as follows:*

$$\varphi_2(H, H; H) + \beta\varphi_1(H, H; H) = \pi_3(H, K, H) + \beta\pi_1(H, K, H).$$

The right-hand term remains constant for any value of H . In fact, taking into account the functional form of π (see equation (18)), we have $\pi_3(H, K, H) =$

$-\delta(K/H)$ and $\pi_1(H, K, H) = \delta(K/H) + (1 - \alpha)(K/H)^\alpha$. These two terms only depend on the ratio K/H . From condition (31) we have $\beta\pi_2(H, K, H) = \beta[1 + \alpha(K/H)^{\alpha-1}] = 1$, which uniquely determines the ratio K/H .

Let us now look at the left-hand term. Because φ is homogenous of degree $\rho > 1$, the derivatives φ_1 and φ_2 are homogeneous of degree $\rho - 1$. Therefore, the left-hand-side term can be written as

$$\varphi_2(H, H; H) + \beta\varphi_1(H, H; H) = [\varphi_2(1, 1; 1) + \beta\varphi_1(1, 1; 1)]H^{\rho-1}.$$

Because $\rho > 1$, this term is strictly increasing in H , converges to zero as $H \rightarrow 0$ and to infinity as $H \rightarrow \infty$. Therefore, there exists a unique value of H that solves this condition. The uniqueness of H then implies the uniqueness of K . *Q.E.D.*

C Proof of Proposition 3

Taking the first order condition in problem (40) and solving for the wage we get:

$$\hat{w}_t = (1 - \eta) [\beta J(h_{t+1}, k_{t+1}) - \hat{D}_f(h_{t+1})] - \eta [\beta W(h_{t+1}, k_{t+1}) - \hat{D}_w(h_{t+1})].$$

After substituting the repudiations values defined in (26) and (38), the wage can be written as:

$$\hat{w}_t = \beta J(h_{t+1}, k_{t+1}) - k_{t+1} - \tau h_{t+1}.$$

Therefore, the ex-post utility of the worker given the investment h_{t+1}^{Old} and k_{t+1}^{Old} is:

$$-\varphi(h_t, h_{t+1}^{Old}) + \hat{w}_t + \beta W(h_{t+1}^{Old}, k_{t+1}^{Old}).$$

Substituting the wage derived above, this can be written as:

$$-\varphi(h_t, h_{t+1}^{Old}) - k_{t+1}^{Old} - \tau h_{t+1}^{Old} + \beta J(h_{t+1}^{Old}, k_{t+1}^{Old}) + \beta W(h_{t+1}^{Old}, k_{t+1}^{Old}). \quad (59)$$

This utility should be compared to the utility received if the worker quits the current employer before the investment in knowledge. In this case the worker would get:

$$\max_{h_{t+1}} \left\{ -\varphi(h_t, h_{t+1}) + \hat{D}(h_{t+1}) \right\}.$$

Substituting the definition of the repudiation value provided in (38), this can be written as:

$$\max_{h_{t+1}, k_{t+1}} \left\{ -\varphi(h_t, h_{t+1}) - k_{t+1} - \tau h_{t+1} + \beta J(h_{t+1}, k_{t+1}) + \beta W(h_{t+1}, k_{t+1}) \right\}, \quad (60)$$

with solutions h_{t+1}^{New} and k_{t+1}^{New} .

The comparison of (59) and (60) makes clear that the value of quitting (equation (60)) is bigger than the value of staying (equation (59)) unless $h_{t+1}^{Old} = h_{t+1}^{New}$ and $k_{t+1}^{Old} = k_{t+1}^{New}$. *Q.E.D.*

D Proof of Proposition 4

In the steady state without commitment, potential new firms start with the same knowledge H as incumbents. Because $H = f(H)$, (45) can be written as:

$$\tau + \varphi_2(H, H; H) = \beta \left[\pi_1(H, K, H) - \varphi_1(H, H; H) \right] + \beta f_1(H) \left[\pi_3(H, K, H) + \tau \right],$$

which determines the steady state knowledge for incumbent and new firms when the investor does not commit (double-sided limited enforcement).

This condition must be compared to the optimality condition that determines the steady state knowledge when the investor commits to the contract (one-side limited enforcement). This is given by equation (32), which we rewrite as:

$$\varphi_2(H, H; H) = \beta \left[\pi_1(H, K, H) - \varphi_1(H, H; H) \right] + \pi_3(H, K, H).$$

The homogeneity of degree ρ of the cost function φ implies that the derivatives are homogeneous of degree $\rho - 1$. Therefore, the above two conditions can be rewritten as:

$$\begin{aligned} \left[\varphi_2(1, 1; 1) + \beta \varphi_1(1, 1; 1) \right] H^{\rho-1} &= \beta \pi_1(H, K, H) + \beta f_1(H) \pi_3(H, K, H) \quad (61) \\ &- \tau \left[1 - \beta f_1(H) \right] \end{aligned}$$

$$\left[\varphi_2(1, 1; 1) + \beta \varphi_1(1, 1; 1) \right] H^{\rho-1} = \beta \pi_1(H, K, H) + \pi_3(H, K, H). \quad (62)$$

Because $\rho - 1 > 0$, the left-hand terms are strictly increasing in H , converge to zero as $H \rightarrow 0$ and to infinity as $H \rightarrow \infty$. We further observe that, as shown in the proof of Proposition 6, the terms π_1 and π_3 only depend on the ratio K/H . This term is uniquely pinned down by condition (27), which is the same for both economies. Therefore, $\pi_1(H, K, H)$ and $\pi_3(H, K, H)$ do not change as H changes.

Consider first the case with zero start-up cost, that is, $\tau = 0$. If $f_1(H) \leq 1$, as postulated in the proposition, the term $\beta f_1(H) < 1$. Because $\pi_3(H, K, H) < 0$ and $\beta f_1(H) < 1$, the right-hand side of (61) is bigger than the right-hand side of (62) for a given H . This implies that the value of H in the first equation must be bigger than in the second, that is, $H^{NC} > H^C$. Without capital obsolescence, $\pi_3(H, K, H) = 0$, and therefore, (61) and (62) are indistinguishable if $\tau = 0$.

Let us now consider the case $\tau > 0$. This variable only affects condition (61). Because $\beta f_1(H) < 1$, an increase in τ reduces the right-hand side of (61), which requires a lower value of H . For a sufficiently large τ , the steady-state level of knowledge declines to the point in which $H^{NC} < H^C$. *Q.E.D.*

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