3

The Fiscal Theory of Money as an Unorthodox Financial Theory of the Firm

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1 Introduction

There are three issues in which everyday macroeconomic life and economic theory seem to be quite apart. The first, is the zeal by which governments in Europe, and elsewhere,

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pursue fiscal discipline as an almost precondition for price stability. One would think that governments were trying to implement a well established theorem in monetary theory, but although the need to coordinate fiscal and monetary policies is a well understood principle[1], such a theorem has been missing.

The second, is the fact that in many rational expectations monetary equilibrium models the price level is indeterminate. A fact that the *Quantity Theory* had unveiled long time ago. In contrast, everyday discussions regarding exchange rate (or other asset prices) movements seem to take prices as being determined.

A third, is that, while in order to avoid indeterminacy problems, many economists, notably Freidman (1959), have advocated for monetary policies that target the supply of money, more and more, Central Banks have been following endogenous monetary policies, for example, targeting nominal interest rates. Raising again the problem of price determinacy.

In this context, the *Fiscal Theory of Money,* (Sims (1994, 1995), Woodford (1995, 1996)) seems to be the missing theory that many had been looking for. It is *a theory* that provides a rationale to the above three "common views." The main elements of the theory are not new. For example, *Real Balance effects* were already accounted by Fisher (1913) and Wicksell (1936) and, in particular, in Patinkin's seminal work (Patinkin, 1965). Similarly, properly specified models have always taken into account the government's budget, although Christ (1979) had to insist on the importance of taking into account the consolidated budget of the government. What is new in the *Fiscal Theory of Money* is that the implications of government's budget accounting and of possible wealth effects are fully worked out in the context of rational expectations models with no special distortions. That is, in models that one would had expected that *Ricardian equivalence* results would had washed out real balance effects. The results on price determination are in striking contrast with standard *Quantity Theory* prescriptions.

The *Fiscal Theory of Money* is based in an important feature that makes governments different from other agents -say, households. The government's monopoly on a nominal asset, namely, money. But, in this important respect, governments are not different from other agents that issue nominal assets -in particular, firms. As a way to expose, and assess, the central elements of the fiscal theory, I develop it and present it in the context of an equilibrium model with firms that use a mix of debt and equity as outside financing. The corresponding *Fiscal Financial Theory of the Firm* also appears in marked contrast with standard *Asset Price Theory.* In the context of the firm, however, it appears very transparent how the theory, as a theory of price determination, relies on letting agents (firms, in our case) make plans that violate their "no default" constraints. This allows for *Real Balance effects,(Real Financial Assets Effects)* based on a peculiar failure of the Modigliani-Miller theorem: fully rational and unconstrained agents do not take debt and equity financing as being equivalent since (unless prices adjust) they may fail to satisfy standard -"no default"- transversality constraints.

I present the model in Section 2. In Section 3, I develop some of the implications of the theory and discuss its relationship with *Asset Price Theory.* In Section 4 I discuss in which sense, the fiscal theory determines prices when firms' policies are endogenous. In particular, I show how the indeterminacy problem is realted to a problem of policy misspecification. Section 5 concludes.

2 A model with representative agents and firms

In this Section I develop a modified version of the Lucas's (1978) model of Asset Prices. There is a representative firm and a representative consumer. The firm can use as source of outside financing a mix of debt and equity. Markets are complete and agents, being fully rational, only have to satisfy their present value constraints. Financial decisions of the firm are not subject to any distortion, such as taxes, or to legal restrictions to issue new debt or equity. Asset demands are well defined since I assume that the consumer, as a proud owner, derives utility from his ownership of the firm. I do not discuss whether this is a reasonable assumption (it may well be). Nevertheless, it should be clear, from the outset, that one can consider an economy without "preferences for ownership" as a limiting case of the economies studied here (as Woodford's (1998a) "cashless economies" are limiting cases of monetary economies). I also simplify the analysis by considering a deterministic model.

Households

Households are represented by an infinitely lived consumer who receives an exogenous stream of income $\{y_t\}$ as well as initial stock of the firm s_0. If the initial total outstanding stock is S_0, its resulting share of the firm is $\theta_0 = \frac{s_0}{S_0}$. In this representative agent economy $\theta_0 = 1$. At any point in time –say, t- the consumer receives from the firm a dividend d_t

per unit of stock (which, unless otherwise stated, and consistent with common and legal practice, I assume to be non-negative). He can sell his stock, $s_t = \theta_t S_t$, and purchase new one, $s_{t+1} = \theta_{t+1} S_{t+1}$, at the current stock price of q_t (in units of consumption). He can also purchase or sell (real) debt issued by the firm, b_{t+1}, which has a (real) return –between periods t and $t+1$- of R_{t+1}. Therefore, the consumer's problem is

$$\max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(\theta_{t+1}) \right]$$

s.t.

$$q_t \theta_{t+1} S_{t+1} + b_{t+1} \leq y_t - c_t + (q_t + d_t)\theta_t S_t + R_t b_t$$

$$\text{and } \lim_{T\to\infty} R_{0,T}^{-1} \left(b_{T+1} + q_T \theta_{T+1} S_{T+1} \right) = 0$$

Where $R_{t,t} \equiv 1; R_{t,T} \equiv R_{t+1} \times \cdots \times R_T$. It is assumed that u and v satisfy standard monotonicity, strict concavity and differentiability properties. Furthermore, $v'(1) > 0$ and $\lim_{c\to 0} u'(c) = \infty$.

Whenever the consumer buys stock, s_{t+1}, it must be that

$$\begin{aligned} \frac{v'(\theta_{t+1})}{u'(c_t)} &= q_t \left(1 - \frac{q_{t+1} + d_{t+1}}{q_t} \frac{1}{R_{t+1}} \right) S_{t+1} \\ &\equiv q_t \left(1 - \frac{R^s_{t+1}}{R_{t+1}} \right) S_{t+1} \equiv q_t \Delta_{t+1} S_{t+1} \end{aligned} \quad (1)$$

where R^s_{t+1} is the return on a unit of stock held from period t to period $t+1$. The consumer also satisfies the standard Euler equation:

$$u'(c_t) = \beta R_{t+1} u'(c_{t+1}) \quad (2)$$

These two conditions, (1) and (2), together with the consumer's budget constraint, characterize the consumer optimal consumption and portfolio decisions.

The intertemporal budget constraint, together with the transversality condition (the present value of his "terminal" portfolio must zero), are equivalent to his present value budget constraint. To see this, let $V_t \equiv (q_t + d_t)\theta_t S_t + R_t b_t$; that is, the value of the household portfolio -of firm's equity and debt- at the beginning of period t after the stock market has cleared. Then, the value of the portfolio has the following law of motion:

$$V_{t+1} = R_{t+1}\left[V_t + y_t - c_t - q_t \Delta_{t+1}\theta_{t+1}S_{t+1}\right]$$

which results in

$$V_0 + \sum_{t=0}^{T} R_{0,t}^{-1}\left(y_t - c_t - q_t \Delta_{t+1}\theta_{t+1}S_{t+1}\right) = R_{0,T+1}^{-1}V_{T+1}$$

or, even more conveniently, in

$$\begin{aligned} & V_0 + (y_0 - c_0) + \sum_{t=1}^{T} R_{0,t}^{-1}\left(y_t + d_t\theta_t S_t + \left(q_t - q_{t-1}R_t\right)\theta_t S_t - c_t\right) \\ = \; & R_{0,T}^{-1}\left(b_{T+1} + q_T\theta_{T+1}S_{T+1}\right) \end{aligned} \quad (3)$$

and, by the transversality condition, the consumer's present value budget constraint is satisfied:

$$V_0 + (y_0 - c_0) + \sum_{t=1}^{\infty} R_{0,t}^{-1}\left(y_t + d_t\theta_t S_t + \left(q_t - q_{t-1}R_t\right)\theta_t S_t - c_t\right) = 0$$

Notice how the consumer's income is composed of external income, y_t, dividends, $d_t\theta_t S_t$, and capital gains, $\left(q_t - q_{t-1}R_t\right)\theta_t S_t$, derived from his portfolio of stock.

Firms

I will not focus on firm's objectives or investment decisions, but, instead, in its financial plans (one can easily imbed the current analysis in a more explicit model with profit maximizing firms). Therefore, the description of the firm is very simple: associated with an investment policy $\{i_t\}$ there is a stream of profits $\{\pi_t\}$. In addition to its investment policy, the firm must decide a dividend policy $\{D_t\}, D_t = d_t S_t$, a stock policy $\{S_t\}$ and a debt policy $\{b_t\}$. Although different units within the firm may decide these policies, such decisions can not be made independently since together they must satisfy the firm's budget constraint. The firm's intertemporal budget constraint is:

$$q_t S_{t+1} + b_{t+1} + \pi_t \geq i_t + q_t S_t + D_t + R_t b_t \tag{4}$$

Such a constraint, of course, does not prevent the firm from running a Ponzi scheme. A minimal requirement is that the present value of its "terminal" liabilities must be zero. If the outstanding liabilities in period t are $W_t \equiv (q_t + d_t)S_t + R_t b_t$ (i.e., W_t is *the value of the firm*), and I define $z_t \equiv \pi_t - i_t$ (i.e., the "primary surplus" of the firm in period t) then (4) can be written as

$$W_{t+1} \geq R_{t+1}\left[W_t - z_t - q_t \Delta_{t+1} S_{t+1}\right] \tag{5}$$

which, without loss of generality, can be assumed to be satisfied as equality; i.e.,

$$W_t = R_{t,T}^{-1} W_T + \sum_{n=t}^{T-1} R_{t,n}^{-1}\left[z_n + q_n \Delta_{n+1} S_{n+1}\right]$$

Then the requirement that the present value of its "terminal" liabilities be zero is

$$\lim_{T\to\infty} R_{0,T}^{-1} W_T = 0 \tag{6a}$$

which guarantees that the current liabilities (the current value of the firm) are equal to the present value of its net revenues. That is,

$$(q_t + d_t)S_t + b_t R_t = \sum_{n=t}^{\infty} R_{t,n}^{-1} \left[z_n + q_n \Delta_{n+1} S_{n+1} \right] \tag{7}$$

Since the firm can raise rents by expanding its stock, it is convenient to express its outstanding liabilities in terms of the price of the stock. That is, let $\omega_t \equiv \frac{W_t}{q_t}$, then its liabilities evolve according to

$$\omega_{t+1} = I_{t+1} \left[\omega_t - \frac{1}{q_t}(z_t + q_t \Delta_{t+1} S_{t+1}) \right] \tag{8}$$

where $I_{t+1} \equiv R_{t+1} \frac{q_t}{q_{t+1}}$. Notice that if I_{t+1} is given, then, and only then, ω_{t+1} is predetermined, before the stock market operates, in period $t+1$.

In summary, the firm present value constraint in period t, (7), can be expressed as

$$q_t \omega_t = \sum_{n=t}^{\infty} R_{t,n}^{-1} \left[z_n + q_n \Delta_{n+1} S_{n+1} \right] \tag{9}$$

Equilibria

I can now define rational expectations equilibria for this economy. I will take as given the endowment sequence, $\{y_t\}_{t=0}^{\infty}$, and the initial conditions: $S_0 > 0, \theta_0 = 1$ and b_0 (unless stated otherwise, I assume $b_0 = 0$). Then a rational expectations equilibrium is a price system, of positive sequences, $(\{R_t\}_{t=1}^{\infty}, \{q_t\}_{t=0}^{\infty})$, an allocation $(\{\pi_t\}_{t=0}^{\infty}, \{i_t\}_{t=0}^{\infty}, \{c_t\}_{t=0}^{\infty})$, and debt-equity $(\{b_t\}_{t=1}^{\infty}, \{S_t\}_{t=1}^{\infty})$ and dividend $\{d_t\}_{t=0}^{\infty}$ policies such that: *i)* it satisfies the resource feasibility constraint: $c_t = y_t + \pi_t - i_t \equiv y_t + z_t$, for all $t \geq 0$, *ii)* given the price system, the consumer's optimization problem is satisfied with $\theta_t = 1$, for all $t > 0$, and *iii)* the firm's investment, debt-equity and dividend policies are consistent with the price system and satisfy the firm's budget constraint (7) at $t = 0$. (alternatively, (4), for all $t \geq 0$ and (6a)).

If I substitute the feasibility constraint in (2) and (1) I see that rational expectations equilibria must satisfy, for $t \geq 0$,

$$R_{t+1} = \beta^{-1} \frac{u'(y_t + z_t)}{u'(y_{t+1} + z_{t+1})} \tag{10}$$

and

$$f(y_t + z_t) \equiv \frac{v'(1)}{u'(y_t + z_t)} = q_t \Delta_{t+1} S_{t+1} \tag{11}$$

which can also be written as

$$f(y_t + z_t) = q_t S_{t+1}(1 - I_{t+1}^{-1}) - d_{t+1} S_{t+1} R_{t+1}^{-1} \tag{12}$$

More precisely, (10) and (11), together with the transversality conditions on portfolios and liabilities, *characterize rational expectations equilibria.*

For further reference, notice that one can substitute (10) and (11) into the firm's budget constraint (9) to obtain

$$q_t \omega_t = \sum_{n=t}^{\infty} \left(\beta^{n-t} \frac{u'(y_n + z_n)}{u'(y_t + z_t)} \right) [z_n + f(y_n + z_n)] \tag{13}$$

3 The contrast between the *Fiscal Financial Theory* and the *Asset Price Theory*

Our simple financial model of the firm illustrates a theorem on the *irrelevance of dividend and stock policies.* It is not regarding the possible neutrality of the debt-equity mix (as in the Modigliani-Miller theorem), but the possible independence of asset prices from dividend and stock policies. It is a simple translation of the a theorem of the irrelevance of "quantitative guided" money supply policies, due to Woodford (1995) (Sims,1994, shows similar irrelevance results).

The Fiscal Theory Irrelevance Proposition

In the economy under study, once the real output sequences $(\{y_t\}, \{z_t\})$ are defined (resulting on the right hand side of (13) being positive; e.g., $y_t \geq 0$, $z_t \geq 0$, $y_t + z_t > 0$), then interest rates, $\{R_{t+1}\}$, and "real" liabilities $\{q_t\omega_t\}$ are determined. If, without loss of generality, the initial conditions are given by: $S_0 = 1$, $b_0 = 0$, then

$$q_0\omega_0 = q_0 + d_0 \tag{14}$$

Now, for arbitrary non-negative sequences $(\{S_t\}_{t=0}^{\infty}, \{D_t\}_{t=1}^{\infty})$, (with $\{D_t\}_{t=1}^{\infty}$ satisfying the minimal restriction that the resulting sequence of prices is non-negative), (14) together with (13) determines q_0, and equation (12) I_1. Equation (8), defining the evolution of the firm's labilities, determines ω_1. By iterating forward this process, one can generate a unique path that, it can easily be seen, defines a *rational expectations equilibrium.*

This irrelevance result seems a paradox at first sight. On the one hand, given initial liabilities ω_0 (alternatively, given the initial dividends, d_0), the price of the stock q_0 -determined by equation (13)- is given by the present value of the firm's net profits, which seems what financial theory should say. On the other hand, one seems free to choose the stock and, in particular, the dividend policies almost independently of the firm's profits, while the *Asset Price Theory* argues that the price of the stock must reflect the present value of the stream of dividends!

Aset Pricing computations

To unveil the mystery is helpful to see a particular example. Let $y_t = y > 0$, and $z_t = z > 0$, for $t \geq 1$, $z_0 = 0$ and $y_0 = y + z$. Given the initial conditions, $S_0 = 1$ and $b_0 = 0$, let the equity and dividend polices be defined as: $S_t = 1$, $t > 0$, and $d_0 = 0$ and, for $t > 0$, $d_t = d$; satisfying $0 \leq d \leq z$.

Notice that, in this case, the equilibrium equations (10) and (11) (for $t \geq 0$) reduce to

$$R_{t+1} = \beta^{-1}$$

$$q_t = f(y+z) + \beta(q_{t+1} + d) \tag{15}$$

while the firm's present value constraints (13) take the form

$$\begin{aligned} q_0\omega_0 &= [f(y+z) + \beta z]/(1-\beta) \\ &\equiv q(z) \end{aligned} \tag{16}$$

and, for $t > 0$,

$$\begin{aligned} q_t\omega_t &= [f(y+z)+z]/(1-\beta) \\ &= q(z)+z \end{aligned} \tag{17}$$

According to the standard *Asset Price Theory* (e.g., Lucas, 1978) one must compute asset prices using forward iteration of (15), i.e.,

$$q_t = [f(y+z)+\beta d]/(1-\beta) + \lim_{n\to\infty} \beta^n q_n$$

and imposing the transversality condition

$$\lim_{n\to\infty} \beta^n q_n = 0 \tag{18a}$$

resulting in

$$q_t = q(d)$$

However, given $d_0 = 0$, unless $d = z$, the transversality condition can not be satisfied. Otherwise, since $\omega_0 = 1$, a dividend plan that does not satisfy the budget constraint (16) could be implemented. It follows that *asset prices are uniquely determined*: $q_t = q(z)$.

According to the *Fiscal Financial Theory of the Firm* (16) determines q_0, and therefore, as with the asset price calculation, $q_0 = q(z)$. But, for $t > 0$, determining q_t through (17), together with the evolution of the firm's liabilities (8) may result in a difference sequence of prices. To see this, notice that, with $S_t = 1$ and $R_{t+1} = \beta^{-1}$, (8) can be written as[2]

$$q_{t+1} = q_{t+1}\omega_{t+1} - d_{t+1} + \beta^{-1}(-q_t\omega_t + q_t + z_t)$$

therefore, in our example, it follows that, for $t > 0$

$$q_t = q(z) + (z - d) + \beta^{-(t-1)}\frac{1-\beta^{(t-1)}}{1-\beta}(z-d) \tag{19}$$

That is, unless $d = z$, $\lim_{n\to\infty} \beta^n q_n = (z-d)\beta/(1-\beta) > 0$. The explosive path of stock prices reflect the fact that the dividend policy is a policy of permanent undistributed profits. The firm satisfies the present value budget constraint (in any period) with identity, reflecting the fact that all the rents are properly accounted for. In other words, according to the *Fiscal Financial Theory of the Firm* there is no problem in that the "Dividend Unit" has made a wrong dividend policy: "the market will discipline the firm". There is a unique price sequence, consistent with such dividend policy, that satisfies the difference equation (15), resulting in positive prices, and satisfying the budget constraint. The price sequence obtained above that, instead of taking (18a) as the terminal condition, takes (13) as the initial condition determining q_0.

3.1 Has the "Fiscal Theory" solved the firms's "default problem"?

I have imposed the (weak) requirement that firm's policies should satisfy the "no Ponzi scheme constraint" (6a), $\lim_{T\to\infty} R_{0,T}^{-1} W_T = 0$. Since $W_t \equiv (q_t + d_t)S_t + R_t b_t$, a policy fails to satisfy $\lim_{T\to\infty} R_{0,T}^{-1}(q_T + d_T)S_T = 0$ if, and only if, it fails to satisfy $\lim_{T\to\infty} R_{0,T}^{-1} R_T b_T = 0$. That is, there is another side to the "Asset Price miscalculation" and this is a "Debt policy miscalculation."

To see how orthodox debt policies can change the price determination results, notice that from (4) I obtain that firm's debts satisfy

$$\begin{aligned} b_t &= R_t^{-1}\left[b_{t+1}-(D_t-z_t)+q_t(S_{t+1}-S_t)\right] \\ &= R_{t,T}^{-1}b_{T+1}+\sum_{n=t}^{T}R_{t-1,n}^{-1}\left[-(D_n-z_n)+q_n(S_{n+1}-S_n)\right] \end{aligned} \tag{20}$$

Therefore, under the "no default constraint"

$$\lim_{T\to\infty} R_{0,T}^{-1}b_{T+1}=0 \tag{21}$$

the current (real) debt liabilities are equal to the present value of the undistributed profits, and of the "seignorage" rents from expanding its stock. That is,

$$b_t=\sum_{n=t}^{\infty}R_{t-1,n}^{-1}\left[-(D_n-z_n)+q_n(S_{n+1}-S_n)\right] \tag{22}$$

i.e.,

$$b_t=R_t^{-1}\left[-(q_t+d_t)S_t+\sum_{n=t}^{\infty}R_{t,n}^{-1}\left[z_n+q_n\Delta_{n+1}S_{n+1}\right]\right]$$

Recall, however, that the firm's present budget constraint, at t (7), is

$$(q_t+d_t)S_t+b_tR_t=\sum_{n=t}^{\infty}R_{t,n}^{-1}\left[z_n+q_n\Delta_{n+1}S_{n+1}\right]$$

and, therefore, it is automatically satisfied, under the "no default constraint," (21). In other words, the equilibrium identity

$$b_t=R_t^{-1}\left[-(q_t+d_t)S_t+q_t\omega_t\right] \tag{23}$$

is already satisfied and can not be used to determine q_t, given d_t.

In the above example, $b_0 = 0$ is predetermined, and, for $t > 0$, (20) results in

$$b_t = \beta^{-(t-2)} \frac{1 - \beta^{(t-1)}}{1 - \beta} (d - z) \tag{24}$$

In other words, by no distributing all profits, the firm becomes a net creditor, even in the long-run. It is also clear that, given $d_0 = 0$, the only dividend policy that it is stationary from period one (i.e., $d_t = d$, $t > 0$) and satisfies (21) is $d_t = z$, resulting in $b_t = 0$.

One can argue, that the firms's policies do not have to be constrained by (21), as long as they are constrained by (6a) *and* consumers are constrained by a "no default constraint" of the form $\lim_{T\to\infty} R_{0,T}^{-1} R_T b_T = 0$. Of course, in our economy, if the representative consumer has to satisfy this additional constraint, in making his consumption and portfolio decisions, then the present value of the "undistributed profits" must be zero *in equilibrium*. There are, however, two problems with this argument. First, it is not clear why consumers should have different borrowing constraints than firms (notice that I have imposed symmetric terminal conditions on consumers and firms). Second, with many -say, J- firms, it is not enough to impose a "no default constraint" of the form: $\lim_{T\to\infty} R_{0,T}^{-1} R_T \sum_{j=1}^{J} b_T^j = 0$, since, obviously, this condition does not imply that, for every j, $\lim_{T\to\infty} R_{0,T}^{-1} R_T b_T^j = 0$. It is enough to consider the case in which firms can hold (debt and equity) portfolios from other firms to see that, unless the later -stricter- condition is required, "asset price miscalculations" may prevail in equilibrium[3].

In summary, there is a sense in which there is no debt default problem. As long as a firm's financial policy guarantees that its value is positive (i.e., the right hand side of (13) is positive and the resulting stock prices are positive), it should be free to borrow or lend and define fairly arbitrary dividend policies. After all, in such a rational expectations world, stock prices will adjust to reflect the "true value" of the firm (i.e., $q_t\omega_t$). For example, if the value of the firm becomes non-positive, then it will be reflected in a stock price collapse. It is in this sense that the stock market provides enough information regarding the financial viability of the firm's policies and there is no need for additional debt constraints.

3.2 The unorthodox failure of the *Modigliani-Miller theorem*

Associated with the "unorthodox" policies (i.e., policies that are designed to satisfy (6a), but not necessarily (21)) there is a *peculiar failure of the Modigliani-Miller theorem.* Households can not perceive the value of the firm as being independent of the debt-equity mix. To see this, I can go back to our previous example (with $y_t = y > 0$, and $z_t = z > 0$, for $t \geq 1$, $z_0 = 0$ and $y_0 = y + z$; initial conditions $S_0 = 1$ and $b_0 = 0$ and a constant stock policy $S_t = 1, t > 0$). Suppose that $d_0 = 0$ and, for $t \geq 1$, $d_t = d < z$. As I have seen, if stock holders were to compute prices according to their stream of dividends (i.e., according to asset price computations), then they would make plans according to $q_t = q(d)$. However, at these prices, the firm's present value constraint (23) is satisfied with equality only if the firm is financing a positive amount of debt; in particular, $b_t = \left(\frac{\beta}{1-\beta}\right)(z-d)$. That is, the undistributed benefits should count as liabilities held by consumers. In other words, as in a *Modigliani-Miller world* the household would properly count the firm's debt as expected payments, being indifferent between the corresponding portfolio $((q_t + d_t), b_t) = \left((q(d) + d), \left(\frac{\beta}{1-\beta}\right)(z-d)\right)$ and the portfolio $((q_t + d_t), b_t) = ((q(z) + z), 0)$. But, as we have seen, with the (d_0, d) dividend policy equilibrium prices are given by (19) with the corresponding level of firms's debt -more precisely, credit- given by (24). Something must be wrong with the household's asset pricing computation to account for the fact that the corresponding portfolio can not be part of an equilibrium.

The mechanism that detects this "violation" is the *Real balance effect,* which can be easily illustrated in our context. With a predetermined level of debt, given by (24), with $(q(d) + d)$ the household, who owns all the firm's liabilities, perceives that its wealth is lower -say, than with q_t given by (19)- and, therefore its demands (not just of the stock) are too low. There will be excess supply of goods at prices $q(d)$, implying that they can not be equilibrium prices. Only the forward sequence $\{q_t\}$, given by (19) satisfies equilibrium restrictions for the dividend policy (d_0, d) , given the initial liabilities $b_0 = 0$ and the exogenous process of firm's surpluses (z_0, z) .

Notice that for the *Real balance effect* to work, in this context, it is crucial that the household does not perceive, as a *Modigliani-Miller believer* would, that the value of the firm is independent of the debt-equity mix. Otherwise, faced with a dividend stream d, it would compute asset prices $q(d)$ as if taking for granted that the firm has the appropriate debt policy in order to satisfy the firm's present value budget. The same reasoning that it would do if faced with dividends z resulting in prices $q(z)$. In other words, the *Real balance effect* can only make its job, of detecting that $q(d)$ are not equilibrium prices, if the household does not believe that associated with a change in dividend policy there is the corresponding change of debt policy that leaves the value of the firm, and of its portfolio, constant. It can not have *Modigliani-Miller beliefs* even if it lives in a world without frictions or distortions, in a world that there is no reason not to be a *Modigliani-Miller believer,* other than the fact that firms are no longer constrained to satisfy the "no default constraint" (21).

In summary, the failure of the Modigliani-Miller theorem is not associated with any of the standard market or firm's financing distortions which are known to invalidate the theorem's result. It is simply that firm's debt policies can be designed without accounting for the "no default constraint." Therefore, paraphrasing Woodford[4], I will call a regime where (22) is satisfied, regardless of the evolution of $\{q_t\}$ (and of $\{R_t\}$, which the firm takes as given) a *Modigliani-Miller policy regime.* In such a regime, different debt-equity policies can not change the value of the firm and the household can maintain *Modigliani-Miller beliefs* in making its consumption and portfolio choices.

4 Has the *Fiscal Theory* solved the indeterminacy problem?

The demands for assets depend on the expected returns and not on their current dividends (or past returns), unless they convey information about future returns. In our model -as in most dynamic equilibrium models- the demands for assets -as such- only depend on expected returns. That is, if in period t, $\widehat{R}^s_{t+1}$ is the expected return on a unit of stock, the demand (1) is given by

$$\frac{v'(\theta_{t+1})}{u'(c_t)} = q_t \left(1 - \frac{\widehat{R}^s_{t+1}}{R_{t+1}}\right) S_{t+1}$$

and in equilibrium it must be that

$$f(y_t + z_t) \equiv \frac{v'(1)}{u'(y_t + z_t)} = q_t \left(1 - \frac{\widehat{R}^s_{t+1}}{R_{t+1}}\right) S_{t+1}$$

and, in this context, rational expectations simply requires that $\widehat{R}^s_{t+1} = R^s_{t+1}$. Nevertheless, rational expectations restrictions may leave the initial price of the stock (or any period price if taken as initial) indetermined. To see this in our model, fix, as before, $z_0 = 0, z_t = z, t \geq 1, b_0 = 0$ and $S_0 = 1$. Now suppose that the firm follows a stock policy aimed at achieving certain asset return R^{s*} ($R^{s*} < \beta^{-1}$). Since $R^s_{t+1} = \frac{q_{t+1}}{q_t} + \frac{d_{t+1}}{q_t}$ there is a manifold of dividend and stock policies that could implement R^{s*}. In particular, let $S_{t+1} = \mu S_t$ and $d_{t+1} = \alpha q_t$, where (μ, α) satisfy: $\frac{1}{\mu} + \alpha = R^{s*}$. Then, using (1) I obtain

$$\frac{q_t}{q_{t-1}} \mu = \frac{q_t S_{t+1}}{q_{t-1} S_t} = \frac{\Delta_t}{\Delta_{t+1}}$$

that is,

$$\mu = \frac{(1 - \alpha\beta)/\pi_t - \beta}{(1 - \alpha\beta) - \beta\pi_{t+1}} \tag{25}$$

If I let $\gamma = \beta/(1 - \alpha\beta)$, then (25) reduces to the following rational expectations equilibrium equation

$$\pi_t = (\mu + \gamma - \mu\gamma\pi_{t+1})^{-1}; \qquad t > 0 \tag{26}$$

Since for $t = 0$ (11) takes the form

$$f(y_0 + z_0) = q_0(1 - \frac{q_1 + \alpha q_0}{q_0}\beta)\mu S_0$$

and $S_0 = 1$, $y_0 + z_0 = y + z$, I obtain that

$$q_0 = \frac{1}{\mu} \frac{f(y + z)}{(1 - \beta(\alpha + \pi_1))} \tag{27}$$

There are two steady state solutions to (26): $(1/\mu, 1/\gamma)$ with corresponding rate of returns $(R^s_\mu, R^s_\gamma) = (R^{s*}, \beta^{-1})$; That is, given our assumption on the targeted rate of return, $R^s_\mu < R^s_\gamma$; .i.e., $\mu > \gamma$.

The *indeterminacy problem* in this context is that there is a continuum of solutions to (26), of equilibrium asset prices, characterized by $\pi_1 > 1/\mu$, (27) and $\pi_t \longrightarrow 1/\gamma$. Does (26), together with (27), fully characterize the set of rational expectations equilibria?

The answer to the last question depends on how firm's policies are defined. Notice that, given initial conditions, I have defined a stock policy $S_{t+1} = \mu S_t$ and an endogeneous dividend policy $d_{t+1} = \alpha q_t$, for $t \geq 0$. To see that, with these policies, in equilibrium budget constraints are also satisfied, notice that now firm's liabilities -i.e., eq. (8)- evolve according to

$$q_{t+1}\omega_{t+1} = \beta^{-1}\left[q_t\omega_t - z_t - (q_t(1-\alpha\beta) - q_{t+1}\beta)\mu^{t+1}\right]$$

and, taking into account that, as before, the firm's present value budget constraints (13) reduce to: $q_t\omega_t = q(z) + z$, for $t > 0$, and $q_0\omega_0 = q(z)$, it follows that asset prices evolve according to

$$q_{t+1} = -\beta^{-1}\left(\frac{1}{\mu}\right)^{t+1} f(y+z) + \frac{1}{\gamma}q_t \tag{28}$$

However, (28) is automatically satisfied whenever (26) and (27) are satisfied. Furthermore, since $R^s_\gamma = .\beta^{-1}$, the transversality conditions are satisfied. In this sense, the *indeterminacy problem* remains even when I account for firm's -and consumer's- budgets. But there is a missing piece in our definition of firm's policies: d_0. That is, our endogenous dividend policy is not completely defined unless I define dividends in period zero. Period zero dividends must, in turn, satisfy the firm's budget constraint. That is,

$$q_0 + d_0 = q_0\omega_0 = q(z) \tag{29}$$

It follows that given a choice of d_0 there is a unique sequence of equilibrium asset prices satisfying (29), (26) and (27); provided that $\pi_1 \geq 1/\mu$. Alternatively, for any sequence of asset prices satisfying (26) and (27), with $\pi_1 \geq 1/\mu$, there is a unique initial dividend d_0 satisfying (29). It is in this sense that the *indeterminacy problem* disappears.

For example, by reverting (28) I can consider again the asset price computation. i.e., for any $T > t$,

$$q_t = \beta^{-1}\left(\frac{1}{\mu}\right)^t f(y+z)\frac{\gamma}{\mu}\frac{1-(\gamma/\mu)^T}{1-(\gamma/\mu)} + \gamma^T q_T$$

and imposing the transversality condition $\lim_{n\to\infty}\gamma^n q_{n+1} = 0$ (the effective discount factor is γ), results in $\pi_t = 1/\mu$, $t > 0$ and

$$q_0 = \beta^{-1} f(y+z)\frac{\gamma/\mu}{1-(\gamma/\mu)} = \frac{1}{\mu}\frac{f(y+z)}{(1-\beta R^{s*})}$$

In other words, the only initial dividend policy consistent with the target R^{s*} being achieved is

$$\begin{aligned} d_0 &= q(z) - \frac{1}{\mu}\frac{f(y+z)}{(1-\beta R^{s*})} \\ &= \left[f(y+z)\left(\frac{\mu-\gamma\beta^{-1}}{\mu-\gamma}\right) + \beta z\right](1-\beta)^{-1} \end{aligned}$$

Notice that the requirement $d_0 \geq 0$ restricts R^{s*} not to be to close to β^{-1}.

4.1 It is not just a question of *transversality conditions*

The previous discussion, and some of the criticisms to the "Fiscal Theory of Money" relies on the firm not satisfying the "no-default" constraint, but this, although revealing, is not a crucial feature. A local version of the above determinacy results can still be satisfied when restrictions on the amount of debt are imposed and, therefore, the "no-default" constraint is satisfied (see, for example, Woodford, 1998).

A simple example can, again, be used to illustrate the point. As before, let $z_0 = 0, z_t = z, t \geq 1, b_0 = 0$ and $S_t = 1, t \geq 0$. But now consider the possibility of "over-distributing" dividends. In particular, let $z \leq d \leq z(1-\beta)^{-1}$. As (19) shows, this may result in negative prices. To avoid such a stock collapse, let $T(d)$ be the largest integer t satisfying

$$(\beta^{-t} - 1)(d - z) \leq z$$

(notice that by assumption $T(d) \geq 1$), and let $d_t = d$, for $t = 1, ..., T(d)$ and $d_t = 0$ for $t > T(d)$, then

$$\begin{aligned} q_{T(d)} &= \overline{q}(z) \equiv f(y+z)/(1-\beta) \\ &= q(z) - \beta z/(1-\beta) \end{aligned}$$

and, iterating on (15), I obtain

$$q_0 = (1 - \beta^{T(d)})q(d) + \beta^{T(d)}\overline{q}(z)$$

that is, the policy of distributing $d \geq z$ for $T(d)$ periods results in asset prices decaying from q_0 to $\overline{q}(z)$ if $d > z$ and in constant prices $q_t = q(z)$ if, and only if, $d = z$. Notice, furthermore, that

$$
\begin{aligned}
q_0 &= f(y+z)/(1-\beta) + (1-\beta^{T(d)})\beta d/(1-\beta) \\
&\approx f(y+z)/(1-\beta) + \beta z/(1-\beta) = q(z)
\end{aligned}
$$

In other words, within the stated bounds, d can be chosen arbitrarily, but if $d_t = d$, for $t = 1, ..., T(d)$ and $d_t = 0$ for $t > T(d)$, then $d_0 \approx 0$. Alternatively, one can choose $d_0 > 0$ and (provided that $\beta z > (1-\beta)d_0$) a d, satisfying $(1-\beta)z \leq (1-\beta)d \leq z - (1-\beta)d_0$, then define the dividend policy $d_t = d$, for $t = 1, ..., T(d, d_0)$ (where $T(d, d_0)$ is the largest integer t such that $\beta(\beta^{-t} - 1)d \leq \beta z - (1-\beta)d_0$) and $d_t = 0$ for $t > T(d, d_0)$. In this case, $q_0 \approx q(z) - d_0$ and $q_{T(d,d_0)} = \overline{q}(z)$.

Notice that $b_t = \overline{b}(z) \equiv \beta z/(1-\beta)$ is the maximum amount of debt consistent with the present value constraint (17) being satisfied and $q_t = \overline{q}(z)$. That is, the above policies can alternatively be stated in terms of a debt ceiling $\overline{b}(z)$, resulting in a dividend period $T(d)$, such that $b_{T(d)} = \overline{b}(z)$.

The question arises on whether one can state the dividend policy in the following terms: "choose $d_0 \geq 0$ and d (satisfying the above restrictions) and let $d_t = d$ for as long as $q_t \geq \overline{q}(z)$ (alternatively, for as long as $b_t \leq \overline{b}(z)$) and $d_t = 0$ thereafter". Notice that, stated like this, one can not make an asset price computation since, in principle, the terminal condition for (15) is undefined. Nevertheless, a rational expectations path (satisfying the present value budget constraint), results in a uniquely defined dividend period $T(d, d_0)$, consistent with such a dividend policy. For example, as I have seen, $d_0 = 0$ and $d_t = z$ results in $T(d) = \infty$.

In this context, the fiscal theory states that the above loose statement of policy is admissible since, by only considering rational expectations equilibrium paths, such a policy results in a unique dividend period $T(d, d_0)$ and a corresponding sequence of asset prices -satisfying (15).

5 Conclusions

I have extended (translated) the *Fiscal Theory of Money* to the problem of outside financing of a firm through debt and equity. Although with this change from governments to firms I have lost some degrees of freedom (the ability of governments to affect economies in ways that firms usually can not), I may have gained in clarity. In fact, since, on the one hand, large firms hold fairly sophisticated portfolios and their revenues far exceed the GDP of some countries, and, on the other hand, governments neither act in isolation nor have unlimited monopoly power, it is far from clear that governments should be treated different from other economic agents -in particular, firms.

I have emphasized how the *Fiscal Theory* works as a theory of price determination, in contrast with the *Asset Price Theory.* In particular, I have stressed how a pre-condition for price determination is that firms are allowed to make plans that do not satisfy their budgets. In such regimes, the Modigliani-Miller theorem fails since the debt-equity mix should be perceived as affecting the value of the firm. Furthermore, since price determination means that *in equilibrium* budgets are satisfied in spite of the "unorthodox" policy design, in a way the theory solves the firms' "default problem." These features, however, more than the strength of the theory, may be showing its weakness. The *Real Balance effects* as price determination mechanisms, in which the theory is based, seem more implausible when considered in a more general setting where many economic agents supply nominal financial assets and households, and firms, hold fairly diversified portfolios. In our model, when households perform asset price calculations resulting in non-equilibrium asset prices, they react with excessively low (or high) demands. Market clearing requires prices given by firms' present value budget constraints as the Fiscal Theory postulates. It is an empirical matter, whether such adjustments take place. There is ample evidence, that bankruptcy problems result in efficiency losses. As we have seen, with the "right prices" (i.e., given by fiscal theory computations), there are no efficiency losses, the market properly prices the -possibly, unreasonable- policies of firms. No need for restrictions on stock policy or other forms of regulation either.

There is also ample experimental evidence that, when there is an indeterminacy problem, taking into account the learning process can help to predict which equilibria are more likely to occur (see, for example, Marimon and Sunder, 1993). In contrast, as we have seen, the fiscal theory "resolves" in many contexts, the indeterminacy problem. In our model of the firm, this is even more clear. The indeterminacy problem is associated with a misspecification of policy, in particular, with a misspecified period zero dividend. Notice that it is not uncommon in Monetary Policy. Nevertheless, the determinacy result requires that proper present value calculations are made by all agents. In our deterministic context this is already difficult, in a stochastic context, as Hansen *et al.* (1991), this may be close to impossible. Adaptive learning agents may make miscalculations, but the final process may still be well defined (even more if policies satisfy "non default" constraints) and, as existing experimental evidence shows, it is unlikely that all the equilibrium paths *determined* by the fiscal theory will arise. This, again, is an empirical matter, but if we had to bet... .

6 Notes

1. See, for example, Sargent and Wallace 1981) *Unpleasant Monetarist Arithmetic.*

2. Or, equivalently, use the fact that (15) can also be written as

$$q_{t+1} = \beta^{-1} q_t + (z - d) - \beta^{-1}(1 - \beta)q(z).$$

3. Notice that I am not allowing for private agents to make plans resulting in unbounded demands -say, by rolling over debts- which, as Woodford (1998b) recognizes, will result in the non-existence of equilibrium.

4. Woodford (1995,1996) calls a regime in which the government policies satisfy the "no default constraint," regardless of the evolution of prices, a *Ricardian policy regime.* In such regime, the Ricardian propostion is satisfied (see, Woodford (1995, 1996) for a further discussion of this issue).

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