On the optimal design of a Financial Stability Fund*

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Abstract

We develop a model of the Financial Stability Fund (Fund), which can be set by a union of sovereign countries. The Fund can improve the countries' ability to share risks, and borrow and lend, with respect to the standard instrument used to smooth fluctuations: sovereign debt financing. Efficiency gains arise from the ability of the Fund to offer long-term contingent financial contracts, subject to limited enforcement (LE) and moral hazard (MH) constraints. By contrast, standard sovereign debt contracts are uncontingent and subject to untimely debt roll-overs and default risk. We quantitatively compare the constrained-efficient Fund economy with the incomplete markets economy with default. In particular, we characterize how (implicit) interest rates and asset holdings differ, as well as how both economies react differently to the same productivity and government expenditure shocks. In our economies, calibrated to the euro area 'stressed countries', substantial welfare gains are achieved, particularly in times of crisis. Our theory provides a basis for the design of a Fund beyond the current scope of the *European Stability Mechanism* (ESM), and a theoretical and quantitative framework to assess alternative risk-sharing (shock-absorbing) facilities, as well as proposals to deal with the euro area 'debt-overhang problem'.

Key words: Recursive contracts, debt contracts, partnerships, limited enforcement, moral hazard, debt restructuring, debt overhang, sovereign funds.

JEL classification: E43, E44, E47, E63, F34, F36

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1 Introduction

> "For all economies to be permanently better off inside the euro area, they also need to be able to share the impact of shocks through risk sharing within the EMU."

This quote from the *Five Presidents' Report* (2015) recognizes a widely accepted fact: without a Federal Budget, or an institutional framework with similar fiscal automatic stabilizers for the euro area, it is unlikely that it will efficiently exploit its capacity for risk sharing, and follow stabilization policies, with the current EMU institutions[1].

We develop a dynamic model of a *Financial Stability Fund* (Fund) as a long-term partnership addressing three features that are usually seen as the most problematic for a risk-sharing institution to be sustainable, when the partnership is a union of sovereign countries. First, sovereignty means that countries can always exercise their right to exit the institution (e.g. defaulting on their obligations), but it also means that risk-sharing transfers should never become permanent transfers or go beyond the level of redistribution that is accepted by all partners. To take this into account, *Fund contracts* are subject to *limited enforcement constraints* (LE), which make the fund stable – namely, there are no defaults – and sustainable, i.e. there are no undesired losses. In particular, our specific design assumes that there are no expected losses at any point in time – the Fund does not provide any redistribution *ex-ante* or *ex-post*.

Second, the Fund must take into account moral hazard problems, since governments may be able to reduce future social and economic risks by implementing policy reforms, but typically fail to do so whenever these reforms have contemporaneous social-political costs. Again, sovereignty places constraints here, since not only may the Fund have limited capacity to fully monitor policy reform efforts but, more importantly, our Fund design respects that national governments have 'ownership' of their policy reforms. To take this into account, *Fund contracts* are based on country-specific risk assessments and subject to *moral hazard constraints* (MH). Given that they are 'experience rated', countries have an incentive to reduce their risk profile before they formalize a Fund contract. Given that these contracts incorporate moral hazard constraints, risk-sharing transfers are combined with 'performance-based' long-term rewards (and punishments), which provide incentives for governments to further pursue risk-reduction policy reforms within the contract. Nevertheless, policy reform

[1] For example, Furceri and Zdzienicka (2015) estimate that the percentage of non-smoothed GDP shocks is 20 percent in Germany, 25 percent in the United States, but 70 percent in the Euro Area (EA) for the period 1978 - 2010. Using their methodology, our estimates suggest an even higher 83 percent of non-smoothed shocks for the EA in the period 1995–2015. Beraja (2016) has performed the counterfactual exercise of regarding the United States as Independent States. He finds, using a 'semi-structural methodology' that, where the employment rate's cross-state standard deviation was 2.6 percent in 2010, had it not been a fiscal union that would have been 3.5 percent.

efforts are not contractable and, accordingly, Fund contracts are not conditional on ex-ante reforms or austerity packages, which, not surprisingly, are usually perceived as a lender's imposition over the borrowing sovereign country.

Third, risk sharing among *ex-ante* equal partners without debt liabilities is relatively easy to design and achieve but, unfortunately, this is not the case among European countries, nor has it ever been in other historical unions. In particular, the euro crisis has left a 'debt-overhang problem' which aggravates the euro area divide. Thus, proposals for a 'shock-absorbing facility' are systematically postponed to a later day of greater convergence (e.g. the *Five Presidents' Report*, 2015), which can result in never-ending procrastination. Our Fund design allows for a greater level of heterogeneity regarding the countries' growth, risk and liability profiles, provided that the latter are sustainable. Moreover, we show that risky defaultable sovereign debts are more sustainable if they are transformed into safe Fund contracts. With such an operation, the Fund balance sheet expands with safe assets, allowing the Fund to issue 'safe bonds'. Thus, the Fund can also play an important role in resolving 'debt-overhang'problems, as well as in creating 'high quality liquid assets' for the union.

In sum, the *Financial Stability Fund* is a *constrained-efficient mechanism* which, by integrating the risk-sharing and crisis-resolution functions, becomes a powerful instrument to prevent and confront crises, and is therefore clearly superior to the standard instrument used to smooth consumption: sovereign (defaultable) debt financing. As a by-product of its ability to transform existing risky liabilities into safe Fund contracts, the Fund can become an important absorber of existing sovereign debts – at least partially – and an issuer of safe assets.

It should be noted that *Limited enforcement* (LE) and *moral hazard* (MH) constraints are *forward-looking* constraints (i.e. the future evolution of the contract is part of the current constraint) and, therefore, standard dynamic programming techniques cannot be applied. We use 'recursive contracts'(see Marcet and Marimon 2019) to obtain and characterize the (constrained) efficient Fund contract. To our knowledge, this is the first paper using this approach to study optimal lending contracts with LE and MH constraints, both qualitatively and quantitatively. Qualitatively, we discuss how the LE and MH frictions interact in determining the risk-sharing properties of the Fund contract, as well as the maximum sustainable level of risk sharing that it can provide. Quantitatively we evaluate how the euro area stressed countries would have performed with the Fund during the recent financial crisis.

On the more practical side, our modelling environment and the characterization of the Fund allocation/contract can be seen as a first attempt to provide theoretical and quantitative foundations for the design of risk-sharing and credit-resolution institutions in federations and unions. For example, for the possible transformation of the *European Stability Mechanism* (ESM), established in 2012, into a fully developed fiscal fund for the Eurozone, not only by incorporating the risk-sharing function – which is now missing in the euro area – but also by having its conditionality based on *ex-post* realisations insead of *ex-ante* 'austerity and/or reform packages', which are often just promises resulting in *ex-post* renegotiations.

Formally, the model of a Financial Stability Fund consists of a contract between a risk-averse, relatively small and impatient borrower (the sovereign country) and a risk-neutral lender (the Fund itself). To assess the efficiency of the Fund, we use as a benchmark an incomplete markets model where sovereign countries issue long-term defaultable debt (IMD) in order to smooth their consumption. In order to have a qualitative and quantitative comparison of the two economies, we 'decentralize' the Fund contract to generate asset holdings and prices that are comparable to those in the IMD economy. Both in the IMD economy with default and in the Fund economy, interest rates may differ from the risk-free rate. The *positive spreads* in the IMD economy reflect the risk of default. Interestingly, the Fund economy only generates *negative spreads*, reflecting mostly the risk that the lender's participation (i.e. his limit for redistribution) constraint is binding.

Our quantitative results are based upon a calibration of the incomplete markets model using data for the period 1980-2015 from the Euro area countries that were most affected by the European sovereign debt crisis (Greece, Ireland, Italy, Portugal and Spain). The calibrated economy provides a reasonably good fit regarding the key variables of interest. In particular, it generates the level of debt, and the statistical properties of the spread (mean, volatility and correlation with output) that are in line with the data. We then solve for the constrained-efficient Fund allocation using the same parameters as in the incomplete markets economy to assess quantitatively how the euro area 'stressed countries'would have performed had they had a Fund contract. In particular, we compare the IMD and the Fund allocations in a number of ways that include comparing the policy functions, contrasting the time path of the economies under the same shocks and examining how the two economies respond to severe shocks. All these comparisons point in the same direction. The Fund is able to provide superior risk sharing (insurance) against shocks through multiple channels. First, it increases the borrowing capacity of the country significantly, smoothing the impact of shocks when they hit through borrowing. Second, the Fund provides state-contingent payments, generating efficient counter-cyclical primary deficits. Third, while default is costly in the two economies both because of direct output losses and because of temporary exclusion from the sovereign debt market, the design of the Fund eliminates default episodes, and, fourth, as a consequence, the borrower does not have to pay any penalties or high spreads on debt whenever borrowing is most desirable. Finally, under our parametrization, financial markets and the constrained-efficient Fund allocation provide similar incentives for policy effort on average, although in a crisis situation the markets require stronger effort, while the opposite happens in normal times.

Quantitatively, we find that the welfare gains of the Fund are very significant: between 3.5 and 5.9 percent in consumption-equivalent terms, depending on the initial state of the economy. The paper then provides a novel decomposition of these welfare gains. We show that the most important sources of welfare gains are the relaxation of the effective borrowing limits, which imply a higher borrowing capacity in the Fund, and the state contingency of payments. In the case of low initial productivity, these two elements constitute about 95 percent of the total welfare gains, with somewhat higher weight on state contingency. For higher levels of productivity, the benefits of avoiding costly default episodes become moderately important, up to 22 percent, while the importance of state contingency is reduced because insurance is less valued for countries with a higher level of output and a lower risk of getting into a crisis.

We are not the first to address how risks could be shared in a monetary union and how to deal with sovereign debt-overhang problems. For example, as an implicit criticism of different proposals to issue some form of joint-liability eurobonds, Tirole (2015) emphasises the asymmetry issue: the optimal (one-period) risk-sharing contract with two symmetric countries is a joint liability debt contract serving as a risk-sharing mechanism, while the optimal contract between two countries with very different distress probabilities is a debt contract with a cap and no joint liability, where the cap depends on the extent of solidarity, which is given by the externality cost of debt default on the lender. With long-term relationships – as they are among sovereign countries that form a union –we show that better contracts can be implemented: the Fund contracts are *constrained-efficient* and they can be implemented as long-term bonds with state-contingent coupons.

In terms of optimal long-term contracts, Atkeson (1991) and Thomas and Worrall (1994) study lending contracts in international contexts. Both of these papers consider only lack of commitment from the borrower's side. Similar to our paper, Atkeson (1991) also considers moral hazard, but with respect to consuming or investing the borrowed funds and not regarding risk-reduction policies. Finally, in a related and contemporaneous work, Müller et al. (2019), study dynamic sovereign lending contracts with moral hazard, with respect to reform policy efforts, and limited enforcement. They provide an interesting characterization and decentralization of the constrained-efficient allocation in a model that, in relation to ours, is more stylised (normal times are an absorbing state) and their debt contracts rely heavily on complex *ex-post* default procedures. In contrast to this paper, none of these contributions have a quantitative focus. Finally, our model of the Fund as a partnership builds on the literature on dynamic optimal contracts with enforcement constraints (e.g. Kocherlakota 1996, Thomas and Worrall 1988, Marcet and Marimon 2019: in fact, our paper is the most developed application of the latter), as well as on the related literature on the decentralization of optimal contracts (e.g. Alvarez and Jermann 2000, Krueger et al. 2008). Our benchmark incomplete markets economy with long-term debt with default, builds on the model of Chatterjee and Eyigungor (2012), who extend the sovereign default models of Eaton and Gersovitz (1981) and Arellano (2008) to long-term debt.

The paper is organized as follows. Section 2 presents the economy with the Fund and with incomplete markets and defaultable long-term sovereign debt. Section 3 shows how to decentralize the Fund contract with state-contingent long term bonds. Section 4 discusses the calibration. Section 5 quantitatively compares the IMD and Fund regimes, concluding with a welfare comparison and showing the ability of the Fund to confront the 'debt overhang' problem. Section 7 summarizes and concludes.

2 The Economy

We consider an infinite-horizon small open economy where the 'benevolent government' acts as a representative agent with preferences for current leisure, $l = 1 - n$, consumption, c, and effort, e, valued by $U(c, n, e) := u(c) + h(1 - n) - v(e)$[2]. The government discounts the future at the rate β, satisfying $\beta \leq 1/(1 + r)$, where r is the risk-free world interest rate and, in general, we will assume the inequality to be strict.

The country has access to a decreasing-returns labor technology $y = \theta f(n)$, where $f'(n) > 0$, $f''(n) < 0$ and θ is a productivity shock, assumed to be Markovian, $\theta \in (\theta_1, ..., \theta_N)$, $\theta_i < \theta_{i+1}$. The country also needs to cover its government expenditures, which are given by $G = G^c + G^d$ – where $\{G^c_t\}^\infty_{t=0}$ is a Markovian process, with $G^c \in \{G^c_1, \ldots, G^c_{N_G}\}$ and transition probability $\pi^{G^c}(G'|s, e)$, and G^d is relatively small, i.i.d. over time, and independent of G^c. That is, government expenditures are, to an extent, endogenous since the current period effort of the government (representative agent) determines the distribution of expenditures next period, with costly higher effort resulting in a better distribution of expenditures.[3] In sum, the current state of the economy is given by $s = (\theta, G^c, G^d)$, with the three components being independent processes, and overall transition

$$\pi(s'|s, e) = \pi^\theta(\theta'|\theta)\pi^{G^d}(G^{d\prime})\pi^{G^c}(G^{c\prime}|G^c, e).$$

Since is an open economy, its current account $\theta(s^t)f\left(n(s^t)\right) - \left(c(s^t) + G(s^t)\right)$, does not need to be balanced period by period. We consider two economies that differ on how the government can access the international capital markets. In the economy with a *Financial Stability Fund* (Fund), the government accesses the markets indirectly, through a contract with the Fund, which, in turn, has direct access to the international capital market. In contrast, in the economy with debt financing, the government has direct access to the international capital market by issuing non-contingent defaultable long-term debt. The Fund contract is based on a country-specific risk-assessment, it is state-contingent and, by design,

[2]We make standard assumptions on preferences. In particular, we assume that $(c, n, e) \in \mathbb{R}^3_+, n \leq 1$, and u, h, v are differentiable, with $u''(x) < 0, h''(x) < 0$ and $v''(x) > 0$.

[3]The introduction of the residual shock G^d, with distribution π^{G^d}, is for technical reasons. As in Chatterjee and Eyigungor (2012), it guarantees robust convergence of our computational procedure under incomplete markets and defaultable debt.

'default free'. Therefore, the Fund has a safe asset in its balance sheet, which should allow it to issue safe bonds in the international capital market. In particular, we show that the fund contract can take the form of a state-contingent asset, which can be priced. Nevertheless, our underlying assumption is that the government cannot access directly the international capital markets by issuing this asset – e.g. the market for such a country-specific asset is probably too thin and the credibility of the government in the international market is likely to be weaker that that of the fund. In other words, the *Financial Stability Fund* acts as an 'intermediary', that transforms risky liabilities into riskless state-contingent assets. We now describe these economies in more detail.

2.1 The Economy with a *Financial Stability Fund* (Fund)

An economy with a *Financial Stability Fund* (Fund) is modelled as a long-term contract between a fund (also called lender), who can freely borrow and lend in the international market, and an individual partner (also called country or borrower), who is the government of the small open economy. The *Fund contract* defines the relationship between the government and the fund. It is a two-sided limited enforcement contract since it endogeneizes that, on the one hand, the government is sovereign and, therefore, can renege the contract and, on the other hand, that from the perspective of the Fund, the contract must be sustainable – i.e. the lender should not have an incentive to renege the contract in favour of another asset in the international financial market.

Furthermore, we assume that the Fund cannot observe the effort of the partner or, simply, that the effort of the government (representative agent) is not contractable. This implies that the long term contract will have to provide sufficient incentives for the country to implement a (constrained) efficient level of effort. In the *Fund contract*, in state $s^t = (s_0, \ldots, s_t)$, the country consumes $c(s^t)$ and the resulting transfer to the Fund , is $\tau(s^t) = \theta f\left(n(s^t)\right) - (c(s^t) + G)$. When $\tau(s^t) < 0$ the country is effectively borrowing.

2.1.1 The Long Term Contract

With *two-sided limited enforcement* and *moral hazard*, an optimal Fund contract is a solution to the following problem:

$$\max_{\{c(s^t),n(s^t),e(s^t)\}} \mathbb{E}\left[\mu_{b,0}\sum_{t=0}^{\infty}\beta^t U(c(s^t),n(s^t),e(s^t)) + \mu_{\ell,0}\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \tau(s^t)\middle| s_0\right] \tag{1}$$

$$\text{s.t.} \quad \mathbb{E}\left[\sum_{r=t}^{\infty}\beta^{j-t}U(c(s^j),n(s^j),e(s^j))\middle| s^t\right] \geq V^a(s_t), \tag{2}$$

$$v'(e(s^t)) = \beta \sum_{s^{t+1}|s^t} \frac{\partial \pi(s^{t+1}|s_t, e(s^t))}{\partial e(s^t)} V^{bf}(s^{t+1}), \tag{3}$$

$$\mathbb{E}\left[\sum_{j=t}^{\infty}\left(\frac{1}{1+r}\right)^{j-t}\tau(s^j)\middle|s^t\right] \geq Z, \tag{4}$$

$$\text{and} \quad \tau(s^t) = \theta(s^t)f(n(s^t)) - c(s^t) - G(s^t), \quad \forall s^t, t \geq 0,$$

with $Z_0 = 0$ and $Z_t = Z$, for all $t > 0$. Note that (μ_{b0}, μ_{l0}) are the initial Pareto weights. They are determined by making the lender's constraint (4) binding in period zero and state s_0. Note that the notation is implicit about the fact that expectations are conditional on the implemented effort sequence as it affects the distribution of the shocks.

Constraints (2) and (4) are the *limited enforcement constraints* for the borrower and the lender, respectively, in state $s^t = (s_0, \ldots, s_t)$. The outside value for the borrower country is the value of facing incomplete asset markets with defaultable debt, denoted by $V^a(s_t)$ in the state of default. We will explain this scenario in more detail in the next subsection.

The outside option of the lender at any $s^t, t > 0$, is $Z \leq 0$[4]. The parameter Z measures the extent of *ex-post* redistribution the Fund is willing to tolerate. Note that, if $Z < 0$ there are states s^t with positive probability, where the Fund is making a permanent loss in terms of life-time expected net present value – i.e. in the international financial market the Fund can find better investment opportunities and if it does not renege it is because it has committed to sustain $Z \leq 0$. Clearly, the level of Z has an important impact on he amount of risk sharing in our environment and it can thus be interpreted as solidarity, as in Tirole (2015). In our benchmark calibration, we assume that $Z = 0$, implying that the lender does not accept any permanent level of *ex-ante* and *ex-post* redistribution. At the same time, the period by period transfers can be positive or negative, still generating risk sharing. Moreover, even if $Z = 0$, the Fund can be superior to other financial mechanisms, since it can still provide risk-sharing and a higher debt capacity to the government. In the next Section we show how Z constraints the paths of Fund transfers and its effect on prices.

Constraint (3) is the *moral hazard* (i.e. incentive compatibility) constraint with respect to the borrower's effort[5], where $V^{bf}(s^{t+1})$ is the value of the Fund contract to the borrower in state s^{t+1}. By imposing equality in (3), we have implicitly assumed that effort is interior, that is $e > 0$.[6] The interpretation of this constraint is standard: the marginal cost of increasing effort has to be equal to the marginal benefit. The latter is measured as the change in life-time utility due to the change in the distribution of future shocks as a result of the increasing

[4]Our characterisation easily generalises to the case that the outside value of the Fund (lender) is state dependent.

[5]Note that we have used the first-order condition approach here, that is, we have replaced the agent's full optimization problem by its necessary first-order conditions of optimality. According to the results of Rogerson (1985), the first-order conditions are also sufficient if the $\pi^{G^c}(G^{c\prime}|G^c, e)$ functions satisfy the monotone likelihood ratio and the convex distribution function conditions. We show in the calibration section that our functional forms restrictions satisfy these requirements.

[6]The appropriate 'Inada' conditions on $v(\cdot)$ and $\pi^{G^c}(G^{c\prime}|G^c, \cdot)$ guarante interiority in our calibration.

effort.

2.1.2 Recursive Formulation

It is known from Marcet and Marimon (2019) and Mele (2014) that we can rewrite the general fund contract problem as a saddle-point problem:[7]

$$
\begin{aligned}
\text{SP} \min_{\{\gamma_{b,t},\gamma_{l,t},\xi_t\}} \max_{\{c_t,n_t,e_t\}} \mathbb{E}\Bigg[& \sum_{t=0}^{\infty} \beta^t \Big(\mu_{b,t} U(c_t,n_t,e_t) - \xi_t v'(e_t) \\
& \qquad + \gamma_{b,t}[U(c_t,n_t,e_t) - V^a(s_t)] \Big) \\
& + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \Big(\mu_{l,t+1}[\theta_t f(n_t) - c_t - G_t] - \gamma_{l,t} Z \Big) \Bigg| s_0 \Bigg] \\
\text{s.t.} \quad & \mu_{b,t+1}(s_{t+1}) = \mu_{b,t} + \gamma_{b,t} + \xi_t \frac{\partial \pi(s_{t+1}|s_t,e_t)/\partial e_t}{\pi(s_{t+1}|s_t,e_t)}, \\
& \mu_{l,t+1} = \mu_{l,t} + \gamma_{l,t}, \text{ with } \mu_{b,0},\ \mu_{l,0} \text{ given,}
\end{aligned}
$$

where $\beta^t \pi(s^t|s_0)\gamma_b(s^t)$, $\beta^t \pi(s^t|s_0)\gamma_l(s^t)$ and $\beta^t \pi(s^t|s_0)\xi(s^t)$ are the Lagrange multipliers of the limited enforcement constraints (2), (4) and incentive compatibility constraint (3), respectively, in state s^t. The above formulation of the problem defines two new co-state variables $\mu_{b,t}$ and $\mu_{l,t}$, which represent the temporary Pareto weights of the borrower and the lender respectively. These variables are initialised by the original Pareto weights, and they become time-variant because of the limited commitment and moral hazard frictions. In particular, a binding participation constraint of the borrower (lender) will imply a higher welfare weight on the the borrower (lender). In addition, the moral hazard friction (whenever $e > 0$ and $\xi > 0$, i.e., whenever the incentive compatibility constraint is binding) implies that the co-state variable of the borrower will be moving up or down depending on the sign of the likelihood ratio $\frac{\partial \pi(s_{t+1}|s_t,e_t)/\partial e_t}{\pi(s_{t+1}|s_t,e_t)}$. In particular, a positive likelihood ratio (which occurs with a low government expenditure) provides a good signal about effort and hence the borrower will be rewarded with a higher temporary Pareto weight.

It turns out that only relative Pareto weights matter for the allocations, and this allows us to reduce the dimensionality of the co-state vector and write the problem recursively by using a convenient normalization. Let $\eta \equiv \beta(1+r) \leq 1$ and normalize the multipliers as follows: $\nu_{i,t} = \gamma_{i,t}/\mu_{i,t}$, for $i = b, l$, $\tilde{\xi}_t = \frac{\xi_t}{\mu_{b,t}}$ and

$$
\varphi_{t+1}(G^c_{t+1}|G^c_t,e_t) = \tilde{\xi}_t \frac{\partial \pi(s_{t+1}|s_t,e_t)/\partial e_t}{\pi(s_{t+1}|s_t,e_t)} = \tilde{\xi}_t \frac{\partial \pi^c(G^c_{t+1}|G^c_t,e_t)/\partial e_t}{\pi^c(G^c_{t+1}|G^c_t,e_t)}, \tag{5}
$$

[7]Following Marcet and Marimon (2019), we only consider saddle-point solutions and their corresponding saddle-point multipliers; that is, given $F(a,\lambda)$, (a^*,λ^*) solves SP $\min_\lambda \max_a F(a,\lambda)$ if and only if $F(a,\lambda^*) \leq F(a^*,\lambda^*) \leq F(a^*,\lambda)$, for any feasible action a and Lagrangian multiplier λ.

where the multiplier $\varphi_{t+1}(G^c_{t+1}|G^c_t, e_t)$ can be positive or negative depending on whether the derivative with respect to effort in the numerator is positive or negative. Then, a new co-state vector can be recursively defined as:

$$x_{t+1}(G^c_{t+1}) = \frac{1+\nu_{b,t}+\varphi_{t+1}(G^c_{t+1})}{1+\nu_{l,t}}\eta x_t, \text{ with } x_0 = \mu_{b,0}/\mu_{l,0} \tag{6}$$

With this normalization, $\nu_{b,t}$ and $\nu_{l,t}$ become the multipliers of the limited enforcement constraints, corresponding to (2) and (4), and φ_t the multiplier of the incentive compatibility constraint, corresponding to (3). Moreover, the state vector for the problem (including the new co-state) is (x, s) and the *Saddle-Point Functional Equation* (SPFE) – i.e. the saddle-point version of Bellman's equation – is given by:

$$\begin{aligned} FV(x,s) = \text{SP} \min_{\{\nu_b,\nu_l,\tilde{\xi}\}} \max_{\{c,n,e\}} x\Big[(1+\nu_b)U(c,n,e) - \nu_b V^a(s) - \tilde{\xi}v'(e)\Big] \\ + \big[(1+\nu_l)(\theta f(n) - c - G) - \nu_l Z\big] \\ + \frac{1+\nu_l}{1+r}\mathbb{E}\big[FV(x',s')\big|s,e\big] \\ \text{s.t.} \quad x' = \frac{1+\nu_b+\varphi(G^{c\prime}|G^c,e)}{1+\nu_l}\eta x, \text{ and} \\ \varphi(G^{c\prime}|G^c,e) = \tilde{\xi}\frac{\partial\pi^c(G^{c\prime}|G^c,e)/\partial e}{\pi^c(G^{c\prime}|G^c,e)}. \end{aligned} \tag{7}$$

Furthermore (see Marcet and Marimon 2019), the *Fund* value function takes the form:

$$\begin{aligned} FV(x,s) = xV^{bf}(x,s) + V^{lf}(x,s), \text{ with} \\ V^{bf}(x,s) = U(c^b(x,s), n^b(x,s), e^b(x,s)) + \beta\mathbb{E}\big[V^{bf}(x',s')\big|s, e^b(x,s)\big], \text{ and} \\ V^{lf}(x,s) = \tau^b(x,s) + \frac{1}{1+r}\mathbb{E}\big[V^{lf}(x',s')\big|s, e^b(x,s)\big], \end{aligned}$$

where $\tau^b(x,s) = \theta f(n^b(x,s)) - G - c^b(x,s)$. The policy functions for consumption and labor defining the Fund contract are given by the first-order conditions of (7). In particular, $c^b(x,s)$ and $n^b(x,s)$ satisfy:

$$u'(c^b(x,s)) = \frac{1+\nu_l(x,s)}{1+\nu_b(x,s)}\frac{1}{x} \quad \text{and} \quad \frac{h'(1-n^b(x,s))}{u'(c^b(x,s))} = \theta f'(n^b(x,s)). \tag{8}$$

These conditions are standard: the borrower's consumption is determined by her endogenous relative Pareto weight and, given that preferences about these decisions are separable, the labor supply is undistorted. The effort policy $e^b(x,s)$ is more complex since the first-order condition with respect to e are given by:

$$(1+\nu_b(x,s))v'(e^b(x,s)) + \tilde{\xi}(x,s)v''(e^b(x,s)) =$$

$$\sum_{s'|s} \frac{\partial \pi(s'|s, e^b(x,s))}{\partial e} \left[\beta \big(1+\nu_b+\varphi(G^{c'}|G^c, e^b(x,s))\big) V^{bf}(x',s') + \frac{1}{1+r} \frac{1+\nu_\ell(x,s)}{x} V^{lf}(x',s') \right]$$
$$+\beta \sum_{s'|s} \pi(s'|s, e^b(x,s)) \tilde{\xi}(x,s) \left[\frac{\partial^2 \pi(s'|s, e^b(x,s))/\partial e^2}{\pi(s'|s, e^b(x,s))} - \frac{\big(\partial \pi(s'|s, e^b(x,s))/\partial e\big)^2}{\pi(s'|s, e^b(x,s))^2} \right] V^{bf}(x',s'). \tag{9}$$

While equation (9) summarizes the social marginal costs and benefits of exerting effort, the incentive compatibility constraint in (3) only accounts for the benefits and the costs from the borrower's point of view. There are two important differences between the two conditions. First, the effect on the value of the lender is also taken into account by (9) with the appropriate weight of $\frac{1}{1+r}\frac{1+\nu_l(x,s)}{x}$. Second, the optimal design of the lending contract also takes into account that by changing effort we are adjusting the tightness of the of incentive compatibility constraint (3). If we substitute (3) into (9) and use the definition of $\varphi(G^{c'} \mid G^c, e)$, the equation above simplifies to the following equality between the 'non-accounted' marginal cost of effort and the 'non-accounted' expected marginal benefit of effort:

$$\begin{aligned}
\text{NMC}(s) &\equiv \tilde{\xi}(x,s) v''(e^b(x,s)) \\
&= \frac{1}{1+r} \sum_{s'|s} \pi(s'|s, e^b(x,s)) \left[\tilde{\xi}(x,s) \eta \frac{\partial^2 \pi(s'|s, e^b(x,s))/\partial e^2}{\pi(s'^b(x,s))} V^{bf}(x',s') \right. \\
&\quad \left. + \frac{1+\nu_l(x,s)}{x} \frac{\partial \pi^G(G'|G, e^b(x,s))/\partial e}{\pi(s'^b(x,s))} V^{lf}(x',s') \right] \equiv \frac{1}{1+r} \mathbb{E}\big[\text{NMB}(s')|s\big].
\end{aligned} \tag{10}$$

Finally, it will be useful to define the primary surplus of the borrower, which is also the transfer to the Fund:

$$\tau(x,s) = \theta f\big(n^b(x,s)\big) - (c^b(x,s) + G). \tag{11}$$

We will study economies where the SPFE equation (7) has a solution for every (x, s) which, in turn, is a recursive *Fund contract* – i.e. a solution of (1) – with the property that, at any (x, s), the *limited enforcement constraints* (2) and (4) cannot be simultaneously binding – i.e. $\nu_b(x, s)$ and $\nu_l(x, s)$ cannot be both positive, – otherwise at (x, s) there would not be expected future rents to be shared and it would be efficient to break the contract; in fact, this would happen if the *limited enforcement constraints* were too tight.[8]

2.2 The Economy with *Incomplete Markets and Default* (IMD)

We now describe the economy with incomplete markets and sovereign debt financing with possible default. This is our benchmark economy, which plays three roles in our analysis. First, with this economy we calibrate to the euro area 'stressed countries'– in other words,

[8]These conditions for existence are easily satisfied in the economies we study. See Marcet and Marimon (2019) for general results on existence.

the *risk assessment* of these countries is done with the IMD model economy. Second, as we discuss below, the default option in the economy with a Fund is the default option in the economy in the IMD economy. Third, we compare this benchmark economy with the economy with a Fund, to assess the value of introducing this fund in the euro area. The incomplete market model with default is a quantitative version of the seminal model by Eaton and Gersovitz (1981) with endogenous labor supply, policy effort, long-term bonds, and an asymmetric default penalty, to achieve a more complete description of the business cycle dynamics of a small open economy with sovereign debt.

With sovereign debt financing, the borrower can issue or purchase *long-term* bonds, which promise to pay constant cash flows across different states. We model long-term bonds in the same way as Chatterjee and Eyigungor (2012). A unit of long-term bond is parameterized by (δ, κ), where δ is the probability of continuing to pay out the coupon in the current period, and κ is the coupon rate. Alternatively, $1 - \delta$ is the probability of maturing in the current period, and this event is independent over time. The coupon rate κ provides a flexible way to capture the coupon payment, where $\delta\kappa$ equals to the expected coupon payment on each unit principal of outstanding debt.

By a purchase of one bond we mean, more precisely, the purchase of one unit of a portfolio of a continuum of bonds of infinitesimal size and the same (δ, κ), but with independent realizations within the portfolio. Thus, one unit of bond (δ, κ) repays $(1 - \delta) + \delta\kappa$ in any given period (as long as the borrower do not decide to default). It also follows that the bond portfolio has a recursive structure, in which only the size of total outstanding debt b matters, regardless of the period in which the bond was issued. Note that δ directly captures the duration of the bond, namely, if $\delta = 0$ and $\kappa = 0$, the bond becomes the standard one-period debt and, in general, the average maturity of the bond equals to $1/(1-\delta)$, which is increasing in δ.

2.2.1 The Budget Constraint and Default Decision

Let b_t denote the size of the bond portfolio (δ, κ) held by the borrower at the beginning of time t. Following the convention in the literature, $b_t > 0$ means holding assets while $b_t < 0$ means having debt. The borrower first makes a decision on whether to default on the promised bond payment of the entire bond portfolio b_t.

No default When the borrower chooses not to default, then the bond payment $(1 - \delta)b_t + \delta\kappa b_t$ will be settled as promised: if $b_t \geq 0$, then the bond payment is part of the borrower's time t income; else if $b_t < 0$, then the borrower will make the required payment to the lender. Choosing not to default allows the borrower to stay in the bond market, so that the borrower may choose the bond holding position b_{t+1} for the next period. The difference between b_{t+1} and the remaining principal δb_t is the net issuance at time t. Due to the recursive structure of the long-term bond, the cash flows starting from $t+1$ onward of both b_{t+1} and δb_t are proportional, and therefore the same unit bond price applies to both. The bond price function $q(s_t, b_{t+1})$ depends on the exogenous shock s_t and the bond position b_{t+1} for the next period. It follows that when the borrower chooses not to default, the budget constraint is as follows:

$$c_t + q(s_t, b_{t+1})(b_{t+1} - \delta b_t) \leq \theta_t n_t^{\alpha} - G_t + (1 - \delta + \delta\kappa) b_t.$$

Default Upon choosing default, the borrower is excluded from the bond market immediately and enters into autarky. As a result, the time t consumption is given by:

$$c_t = \theta^p(\theta_t) f(n_t) - G_t.$$

There are several costs from defaulting. First, there is exclusion from the bond markets that lasts for a random number of periods. If the borrower was excluded from the market in the previous period, then with probability $\lambda < 1$ the borrower regains access to the bond market in the current period, and with remaining probability $1 - \lambda > 0$ the borrower stays in autarky. Moreover, upon regaining access to the bond market, the borrower starts from a zero bond position.

Besides the exclusion from the bond market, the borrower also suffers from a productivity penalty $\theta^p(\theta)$ in autarky. As in Arellano (2008), we assume that the penalty is *asymmetric*, in the sense that the higher productivity is the higher the penalty is (weakly). An asymmetric penalty is crucial for the quantitative performance of models with sovereign debt and default. When the penalty is properly specified, it creates incentives for the borrower to borrow more in good states while deterring default temptation by harsh punishment, and these high levels of debt then induce the borrower to choose default in bad states where the penalty is lower.

2.2.2 Recursive Formulation

Let b be the size of the long-term bond portfolio held by the borrower at the beginning of a period[9] and (s, b), $s = (\theta, G^c, G^d)$, be the state. Let $V_n^{bi}(b, s)$ denote the value function of the borrower in the incomplete market economy at the beginning of a period, when the borrower chooses *not* to default. Then it satisfies

$$V_n^{bi}(b, s) = \max_{c,n,e,b'} U(c, n, e) + \beta \mathbb{E}\left[V^{bi}(b', s') \middle| s, e\right] \tag{12}$$
$$\text{s.t.} \quad c + G + q(s, b')(b' - \delta b) \leq \theta f(n) + (1 - \delta + \delta\kappa) b,$$

where, taking into account that default may occur next period,

$$V^{bi}(b, s) = \max\left\{V_n^{bi}(b, s), V^{ai}(s)\right\}, \tag{13}$$

[9] We assume that $b \in \mathcal{B} = [b_{\min}, b_{\max}]$, with $-\infty < b_{\min} < 0 \leq b_{\max} < \infty$., where we will choose $b_{\min}$ and $b_{\max}$ so that in equilibrium the bounds are not binding.

and $V^{ai}(s)$ is the value of autarky upon default, given by

$$V^a(s) = \max_{n,e} \left\{ u(\theta^p(\theta) f(n) - G) + h(1-n) - v(e) \right\} \tag{14}$$
$$+ \beta \mathbb{E}\left[(1-\lambda) V^a(s') + \lambda V_n^{bi}(0, s') \middle| s, e \right],$$

where λ is the probability to come back to the market and be able to borrow again. Recall that $V^{ai}(s)$ is also the outside option that the borrower considers in the Fund contract when she contemplates whether to default or not.

The optimality condition with respect to effort takes the following form:

$$v'(e) = \beta \sum_{s'|s} \frac{\partial \pi(s'|s,e)}{\partial e} V^{bi}(b', s'). \tag{15}$$

This equation has a similar form as the incentive compatibility constraint (3) and the same interpretation. Moreover, this condition implies that the optimal effort decision only depends on b through b', hence we can write the policy function as $e\,(s, b')$. This simplifies considerably the pricing equation of the bond and consequently our computations.

The bond price also has a recursive structure. Let the default decision be given by:

$$D(s,b) = 1 \text{ if } V^{ai}(\,s) > V_n^{bi}(b,\,s) \text{ and } 0 \text{ otherwise.}$$

The expected default rate is $d(s,b') = \mathbb{E}\left[D(s',\, b') \mid s, e(s,b')\right]$ and the equilibrium bond pricing function $q(s,b')$ satisfies the following recursive equation:

$$q(s,b') = \frac{\mathbb{E}\left[(1 - D(s',b')) \left[(1-\delta) + \delta \left[\kappa + q(s',\, b''(s',\, b')) \right] \ \right] \mid \ s, e(s,b') \right]}{1+r},$$

which can also be expressed as:

$$q(s,b') = \frac{(1-\delta)+\delta\kappa}{1+r} \left(1 - d(s,b')\right) + \delta \frac{\mathbb{E}\left[(1 - D(s',\, b'))\, q(s',\, b''(s',\, b') \mid \ s, e(s,b') \right]}{1+r} \tag{16}$$

Note that, for a one-period bond ($\delta = 0$), this would reduce to the more familiar expression $q(s,b') = \frac{1-d(s,b')}{1+r}$. Note also that, for an outstanding long term bond portfolio of size b, its cash flow stream is given by $(1-\delta)b + \delta\kappa b,\ \delta(1-\delta)b + \delta^2\kappa b,\ \ldots$. Thus, when there is no default, the price q of a unit of a *riskless* long-term bond (δ, κ), given a constant one period discount rate r, is:

$$q = \sum_{t=0}^{\infty} [(1-\delta) + \delta\kappa] \frac{\delta^t}{(1+r)^{t+1}} = \frac{(1-\delta)+\delta\kappa}{r+1-\delta}.$$

This price (tautologically) implies the following one period risk risk free return and implicit intertemporal discount factor:

$$r = \frac{(1-\delta)+\delta\kappa}{q} - (1-\delta)$$

$$Q = \frac{1}{1+r} = \frac{q}{(1-\delta)+\delta\kappa+\delta q}.$$

Similarly, the implied interest rate and intertemporal discount factor of the defaultable long-term bond is given by:

$$r^i(s,b') = \frac{(1-\delta)+\delta\kappa}{q(s,b')} - (1-\delta) \tag{17}$$
$$Q\left(s,b'\right) = \frac{1}{1+r\left(s,b'\right)} = \frac{q(s,b')}{1-\delta+\delta\kappa+\delta q\left(s,b'\right)}$$

resulting as usual in a *positive spread* $r^i\left(s,b'\right)-r \geq 0$, which is strictly positive if $d(s',b)>0$ for some s'.

The optimal policies when there is no default $(c(s,b), n(s,b), b'(s,b), e(s,b'(s,b))$ and those when there is default $(n^a(s), e^a(s))$ are standard dynamic programming solutions to (12) and (14), respectively, whereas the bond price $q\left(s,b'\right)$ and implied interest rate $r\left(s,b'\right)$ are a solution to (16) and (17) respectively. Finally, in order to keep track of debt flows and in order to compare with a counterpart for τ in the Fund contract, it will be useful to define the primary surplus of the borrower, which is also the transfer to the lender, as:

$$\tau^i(s,b) = \theta f\left(n(s,b)\right) - \left(c(s,b)+G\right) = q(s,b')(b'-\delta b) - (1-\delta+\delta\kappa)b. \tag{18}$$

In essence, if the country consumes more than it produces, $\tau^i\left(s,b\right)<0$, we say that the country is running a deficit, whereas the country runs a surplus if if consumes less than it produces, $\tau^i(s,b)>0$. In this sense, we will call $\tau^i\left(s,b\right)$ primary surplus (or primary deficit if negative). Here, it is important to note that, in our economy, taxes (and transfers) are implicitly defined by $Y-C$. This implies that (18) indeed defines both the primary surplus of the government and the net exports. The two key assumptions behind this equivalence are that only the government has access to any inter-temporal borrowing/saving technology and we do not have physical capital accumulation in our model. In our calibration, we will use net exports as the data counterpart of τ^i.

3 Decentralization of the Fund Contract

We now show how to decentralize the optimal Fund contract as a competitive equilibrium with endogenous borrowing constraints. This will allow us to compare the fund contract more directly with the debt contract of the economy with sovereign debt. To do this, we build on the work of Alvarez and Jermann (2000) and Krueger et al. (2008), but we consider long term *state-contingent bonds (assets or securities)* to make it more comparable with the incomplete market model.

3.1 Financial Assets

At the beginning of a period, in state s, the borrower holds a portfolio a of securities (δ, κ), where a fraction $1-\delta$ of the portfolio matures in the current period and a fraction δ pays a coupon κ. The borrower can trade in S securities $a(s')$ with a unit price of $q(s'|s)$ that pay only if state s' is realized next period. The budget constraint is:

$$c + \sum_{s'|s} q(s'|s)\left(a(s') - \delta a\right) \leq \theta(s) f(n) - G(s) + (1 - \delta + \delta\kappa)\, a$$

To make the model as comparable as possible to the IMD economy, we note that the state contingent portfolio can be decomposed into a common 'bond' a' that is carried to the next period and is independent of the next period state, traded at the implicit bond price $q(s) = \sum_{s'|s} q(s'|s)$, and an insurance portfolio of S assets $\hat{a}(s')$, with $a(s') = a' + \hat{a}(s')$, $a' = \left[\sum_{s'|s} q(s'|s)a(s')\right]/q(s)$ and $\sum_{s'|s} q(s'|s)\hat{a}(s') = 0$. The budget constraint can then be rewriten as:

$$c + q(s)\left(a' - \delta a\right) + \sum_{s'|s} q(s'|s)\hat{a}(s') \leq \theta(s) f(n) - G(s) + (1 - \delta + \delta\kappa)\, a$$

Note that other forms of decentralization are possible – for example, using an active management of the debt maturity structure and partial forms of default to induce state contingent contracts, as in Dovis (2016). However our main purpose here is to have clear comparison between the two regimes and this decentralization is possibly the simplest one, since (a, a') can be identified with (b, b') in state s, while $\hat{a}(s')$ corresponds to the additional insurance component provided by the special Arrow security that pays one unit of the long term 'bond' in state s'.

3.2 The Recursive Competitive Equilibrium (RCE)

With the above financial structure we can characterize the equilibrium in the economy with the Fund as a *recursive competitive equilibrium* (in strict sense, a partial equilibrium since the world interest rate is given). In this formulation, the borrower has access to long term state-contingent assets and solves the following dynamic programming problem:[10]

$$W^b(a, s) = \max_{\{c,n,e,a(s')\}} U(c, n, e) + \beta \mathbb{E}\left[W^b(a(s'), s')\middle| s\right] \tag{19}$$

$$\text{s.t. } c + \sum_{s'|s} q(s'|s)(a(s') - \delta a) \leq \theta(s) f(n) - G(s) + (1 - \delta + \delta\kappa)a,$$

$$a(s') = a' + \hat{a}(s') \geq A_b(s'),$$

[10] Note that the borrower, as well as the lender, are modelled as representatives of a continuum of homogeneous borrowers and homogeneous lenders of the same size.

where a', $\hat{a}(s')$ and $q(s'|s)$ are defined as above and $A_b(s')$ is an endogenous borrowing constraint that is given by:

$$W^b(A_b(s'), s') = V^a(s'). \tag{20}$$

Note that, in contrast with the incomplete markets economy, $q(s'|s)$ is independent of the amounts of securities being traded. This follows from the fact that the endogenous borrowing constraint $A_b(s')$ prevents the borrower from defaulting along the equilibrium path. Similarly to the incomplete market case, effort is determined by the following condition:

$$v'(e) = \beta \sum_{s'|s} \frac{\partial \pi(s'|s, e)}{\partial e} W^b(a'(s'), s'). \tag{21}$$

The lender (i.e. the Fund), who has linear preferences for – possibly, negative – consumption solves the following problem:

$$\begin{aligned} W^l(a, s) = \max_{\{c_l, a(s')\}} & \; c_l + \frac{1}{1+r} \mathbb{E}\big[W^l(a_l(s'), s') \big| s, e\big] \\ \text{s.t. } & \; c_l + \sum_{s'|s} q(s'|s)(a_l(s') - \delta a_l(s)) = (1 - \delta + \delta\kappa) a_l(s), \\ & \; a_l(s') \geq A_l(s'), \end{aligned} \tag{22}$$

where the borrowing constraint is given by:

$$W^l(A_l(s'), s') = Z. \tag{23}$$

We assume, that $a(s_0) = -a_l(s_0)$, the initial asset holdings of the borrower and the lender, are given.

The *recursive competitive equilibrium* is defined as follows: (i) Given value functions for the outside value options of the borrower, $V^a(s')$, and of the lender, Z (which could also depend on s), and asset prices $q(s'|s)$ such that (ia) the policy functions $c(a, s)$, $n(a, s)$, $e(a, s)$, $a(s')$, together with the value function $W^b(a, s)$, solve the borrower's problem (19) with the endogenous limit (20), and (ib) the policy functions $c_l(a_l, s)$, $a_l(s')$, together the value function $W^l\,(a_l, s)$, solve the lender's problem (19) with the endogenous limit (23); (ii) the product and labour markets clear, in particular $c(a, s) + c_l(a_l, s) = \theta(s) f(n(a, s)) - G(s)$; and (iii) the asset markets clear, $a(s') + a_l(s') = 0$.

We only consider economies where $W^b(0, s_0) > V^a(s_0)$. In other words, we assume that the outside value options of the borrower and the lender, $V^a(s')$ and Z, are such that a *recursive competitive equilibrium*, with these restrictions, exists – as it is the case in the economies that we calibrate.

3.3 The Fund Contract as an Asset of a RCE

We now show how the Fund contract can be decentralized in a recursive competitive equilibrium with long-term assets and borrowing limits. This allows us to obtain asset prices and holdings supporting the Fund contract, which we can compare to the debt prices and holdings of the incomplete markets economy.

Let $c^*(x,s)$, $n^*(x,s)$, $e^*(x,s)$, and $\tau^*(x,s)$ be the optimal policy allocations of the Fund. We will show that we can construct prices $q^*(s'|s)$ and asset holdings $(a^*(s'), a_l^*(s'))$ such that these and the fund allocations are a competitive equilibrium under the endogenous borrowing limits that satisfy (20) and (23). First, taking into account that in Fund contract under study both *limited enforcement constraints* cannot be simultaneously binding, we can use the allocations to define the price for the state contingent long-term assets as follows:

$$q^*(s'|s) = \frac{1}{1+r}\pi(s'|s, e^*(x,s))\frac{u'(c^*(x',s'))\eta}{u'(c^*(x,s))}\left[1-\delta+\delta\kappa+\delta\sum_{s''|s'} q^*(s''|s')\right]$$

$$\text{if } \nu_b(x',s') = 0 \text{ and } \nu_l(x',s') \geq 0,$$

$$q^*(s'|s) = \frac{1}{1+r}\pi(s'|s, e^*(x,s))\left[1-\delta+\delta\kappa+\delta\sum_{s''|s'} q^*(s''|s')\right]$$

$$\text{if } \nu_l(x',s') = 0 \text{ and } \nu_b(x',s') > 0,$$

while the the price of a long term bond is equal to $q^*(s) = \sum_{s'|s} q^*(s'|s)$. Using the optimality conditions in the fund, the long term asset price can be rewriten as a function of the fund allocations as follows:

$$q^*(s'|s) = \frac{\pi(s'|s, e^*(x,s))}{1+r}\left[(1-\delta+\delta\kappa)+\delta q^*(s')\right]\max\left\{\frac{1+\nu_l(x',s')}{1+\nu_b(x',s')}\frac{1}{1+\frac{\varphi(s'|x,s)}{1+\nu_b(x,s)}}, 1\right\}$$

We also define the intertemporal discount factor:

$$Q^*(s'|s) = \frac{q^*(s'|s)}{(1-\delta+\delta\kappa)+\delta q^*(s')} = \frac{\pi(s'|s, e^*(x,s))}{1+r}\max\left\{\frac{1+\nu_l(x',s')}{1+\nu_b(x',s')}\frac{1}{1+\frac{\varphi(s'|x,s)}{1+\nu_b(x,s)}}, 1\right\}$$

and $Q^*(s) = \sum_{s'|s} Q^*(s'|s)$.

Next, we use the intertemporal budget constraints to construct the *asset holdings* that make the consumption allocations in the optimal contract satisfy the present value budget under these prices, namely:

$$\begin{aligned} a(s^t) &= \sum_{n=0}^{\infty}\sum_{s^{t+j}|s^t} Q^*(s^{t+j}|s^t)\left[c^*(s^{t+j}) - \left(\theta(s^{t+j})f(n^*(s^{t+j})) - G(s^{t+j})\right)\right] \qquad (24) \\ &= -\sum_{n=0}^{\infty}\sum_{s^{t+j}|s^t} Q^*(s^{t+j}|s^t)\tau^*(s^{t+j}) \end{aligned}$$

$$a_l\left(s^t\right)=\sum_{n=0}^{\infty}\sum_{s^{t+j}|s^t}Q^*\left(s^{t+j}\middle|s^t\right)\tau^*\left(s^{t+j}\right)=-a_b\left(s^t\right), \tag{25}$$

where

$$Q^*\left(s^{t+j}\middle|s^t\right)=Q^*\left(s^{t+j}\middle|s^{t+n-1}\right)Q^*\left(s^{t+n-1}\middle|s^{t+n-2}\right)\cdots Q^*\left(s^{t+1}\middle|s^t\right).$$

As for the borrowing constraints, if the *limited enforcement constraint* is binding for an agent in the Fund, we define the borrowing limit for that agent in the decentralized economy to be equal to the corresponding asset holding:

$$A_b\left(s^t\right)=\sum_{n=0}^{\infty}\sum_{s^{t+j}|s^t}Q^*\left(s^{t+j}\middle|s^t\right)\left[c^*\left(s^{t+j}\right)-\left(\theta(s^{t+j})f\left(n_b^*\left(s^{t+j}\right)\right)-G\left(s^{t+j}\right)\right)\right],\ \text{and} \tag{26}$$

$$A_l\left(s^t\right)=\sum_{n=0}^{\infty}\sum_{s^{t+j}|s^t}\left(\frac{1}{1+r}\right)^t\tau^*\left(s^{t+j}\right). \tag{27}$$

Given our assumptions, there is clearly a one-to-one correspondence between the state variable x in the Fund problem and a in the decentralized problem, given by:

$$u'(c(a,s))=\frac{1+\nu_l(x,s)}{1+\nu_b(x,s)}\frac{1}{x}$$

that is, if at s, a and x satisfy this one-to-one correspondence, then $c(a,s)=c^*(x,s)$, $n(x,s)=n^*(x,s)$ and $c_l\,(a,s)=\tau^*(x,s)$. Furthermore, if, following the same one-to-one correspondence, we let $W^b(a,s)=V^{bf}(x,s)$ and $W^l\,(a,s)=V^{lf}\,(x,s)$, then, by construction, the endogenous borrowing limits (20) and (23) are satisfied and they are binding if, and only if, they are binding in the Fund contract.

Similarly, the incentive compatibility condition determining the effort level in the Fund (3) and the first-order condition determining the effort in the RCE (21) are the same – therefore, the effort is also the same and the asset prices $q(s'|s)=q^*(s'|s)$ support agents' competitive policies – including $a'\,(a,s)$ and $a_l'\,(a,s)$, given by (24) and (25). It follows that $W^b(a,s)=V^{bf}(x,s)$ and $W^l\,(a,s)=V^{lf}\,(x,s)$ are value functions of the *recursive competitive equilibrium.*

In sum, the *Fund contract*, with initial Pareto weights (μ_{b0},μ_{l0}) making the lender's constraint (4) binding in period zero and state s_0, can be 'decentralized' as a *recursive competitive equilibrium* with endogenous borrowing constraints.[11]. This 'decentralization' allows us to compare prices and asset allocations in the economy with the Fund and in the economy with incomplete markets and default (IMD). In particular, the implicit interest rate in the decentralized economy can be obtained from the price of the long term bond:

$$r^*(s)=\frac{1}{Q^*(s)}-1,$$

[11]See the Appendix for a discussion of the (constrained) efficiency of the RCE.

which results in a possibly *negative spread*, $r^*(s) - r \leq 0$, since $Q^*(s) \geq \frac{1}{1+r}$.

To understand the negative spread, consider first the case with *no moral hazard*, $\varphi(s'|x,s) = 0$. Looking at the expression for $Q^*(s'|s)$, it is clear that the *negative spread* in this case reflects the fact that the lender's intertemporal participation constraint is binding for some state tomorrow, that is, $Q^*(s) > \frac{1}{1+r}$ only if $\nu_l(x', s') > 0$ for some s'. In that state, which typically occurs when the borrower's Pareto weight, x', is relatively high and the state, s', is bad, and, given this, the borrower's liabilities are in risk to become permanent transfers (i.e. the Fund is in danger of making permanent losses). In this case, the negative spread discourages the Fund from lending since the lender is better off lending (saving) at the riskless interest rate r in the international market – i.e. the negative spread indirectly imposes a constraint on the amount of insurance the borrower can get.

Consider now the case with *moral hazard*. Even if the the lender's intertemporal participation constraint is not binding, we can still have a negative spread since $\varphi(s'|x,s)$ can be negative. Recall that the relative Pareto weight of the borrower is increasing in $\varphi(s'|x,s)$, which is a way for the fund to reward the borrower when he exerts a high level of effort, increasing the likelihood of a good state s' (low government expenditure shock). Alternatively, the lender must discourage the realization of bad states states s'. In other words, in certain states (s, a), negative spreads enforce this behaviour of the lender in a decentralized economy and, therefore, also help to discipline the borrower to exert the right level of effort. In sum, *the negative spread,* $r^*(s) - r < 0$, reflects *the wedge* that aligns the market price with the lender unwillingness to lend in some states of the future.

Paralleling the definition of the primary surplus in the *incomplete markets* economy, the *primary surplus* – or *primary deficit* if negative – in the decentralized fund economy is given by:

$$c_l^*(a,s) = q(s)(a(s') - \delta a) - (1 - \delta + \delta k)\, a = \theta f\,(n\,(a,s)) - (c\,(a,s) + G).$$

4 Calibration

4.1 Functional Forms, Processes and Parameter Values

We use the IMD economy with defaultable debt to calibrate our model for the Euro Area 'stressed countries' during the euro crisis: Portugal, Ireland, Italy, Greece and Spain. The model period is assumed to be one year. The utility of the borrower is additively separable in consumption, leisure and effort. In particular, we assume that $u(c) = \log(c)$, $h(1-n) = \gamma \frac{(1-n)^{1-\sigma}-1}{1-\sigma}$ and $v(e) = \omega e^2$ so that:

$$U(c,n,e) = \log(c) + \gamma \frac{(1-n)^{1-\sigma} - 1}{1-\sigma} - \omega e^2$$

The preference parameters (σ, γ) are set to $\sigma = 0.6887$ and $\gamma = 1.4$. These are used to match the average hours, together with the volatility of consumption relative to GDP. The effort cost parameter is set to $\omega = 0.1$, and is explained below. The risk free interest rate is set to $r = 2.48\%$, the average short-term real interest rate of Germany.

In both economies, the parameters of the long term bond (δ, κ) are set to $\delta = 0.814$ and $\kappa = 0.083$ to match the average maturity and the average coupon rate (coupon payment to debt ratio) of long term debt. After a country defaults in the IMD economy, it faces exclusion for a random number of periods, and the probability that it comes back to the market with sovereign debt upon default is set to be $\lambda = 0.15$. In the Fund economy, the participation constraint of the lender is set to $Z = 0$, implying no expected permanent transfers between the borrower and the lender at any time or state. In other words, the Fund is not build on an assumption of solidarity which would require permanent transfers.

We assume that *Fund-exit is irreversible*, with the interpretation that the fund can commit to exclusion of the borrower. In that case, we assume that the country has the same probability of coming back to the sovereign debt markets as in the IMD economy, hence it is in the same situation as a defaulting country in the IMD economy. If a country defaults, it is also subject to an asymmetric default penalty of the form:

$$\theta^p = \begin{cases} \bar{\theta}, & \text{if } \theta \geq \bar{\theta} \\ \theta, & \text{if } \theta < \bar{\theta} \end{cases} \quad \text{with } \bar{\theta} = \psi \mathbb{E}\theta,$$

where $\psi = 0.8099$. It is known from Arellano (2008), that this asymmetric default penalty is crucial to obtain a significant default probability (and consequently spread) in an economy with defaultable debt. The latter two parameters (λ, ψ), together with the discount factor $\beta = 0.945$ are chosen to match jointly the average debt to GDP ratio, spread level and spread volatility in our sample. Note that this implies a different discount factor for the lender of $\frac{1}{1+r} = 0.9758$, as well as a growth rate for the relative Pareto weight of the borrower of $\eta = 0.9684$ in the optimal contract. The fact that the borrower is less patient than the lender implies that the borrower will tend to get indebted in both economies. As it is well known, in the absence of any frictions (limited commitment or moral hazard) consumption of the borrower would converge towards zero in the long run.

Regarding the technology, we assume that $f(n) = n^\alpha$ with the labor share of the borrower set to $\alpha = 0.566$ to match the average labor share across the Euro Area 'stressed' countries. Table 1 summarizes the parameter values.

Table 1: Parameter Values

α	β	σ	γ	r	λ	ψ	δ	κ	ω	Z
0.566	0.945	0.6887	1.4	0.0248	0.15	0.8099	0.814	0.083	0.1	0

The log of labor productivity, $\log \theta$, is assumed to be a Markov regime switching (MRS)

AR(1) process. In our calibration, we fit the labor productivity $\log(\theta_{it})$ of the five countries to the following panel MRS AR(1) model:[12]

$$\log\theta_{it} = (1-\rho(\mathscr{s}_{it}))\mu(\mathscr{s}_{it}) + \rho(\mathscr{s}_{it})\log\theta_{it} + \sigma(\mathscr{s}_{it})\varepsilon_{it},$$

where $\mathscr{s}_{it} \in \{1,\ldots,R\}$ denotes the regime of country i at time t, $\mu(\mathscr{s}_{it})$, $\rho(\mathscr{s}_{it})$, and $\sigma(\mathscr{s}_{it})$ are the regime-dependent parameters of the process, and $\varepsilon_{it} \overset{\text{iid}}{\sim} N(0,1)$. The country specific regime s_{it} is independent in the cross-section, and follows a Markov chain over time, with an $R\times R$ regime transition matrix P. Since our model does not have any capital accumulation, we use the time series for the labor productivity θ_{it} for the five Euro Area 'stressed countries'. The estimated parameters of the Markov Switching Process are displayed in Table 2. Finally,

Table 2: Parameters of the labor productivity process

	$\mu(\mathscr{s})$	$\rho(\mathscr{s})$	$\sigma(\mathscr{s})$	P	$\mathscr{s}=1$	$\mathscr{s}=2$	$\mathscr{s}=3$
$\mathscr{s}=1$	6.35	0.93	0.02	$\mathscr{s}=1$	0.90	0.10	0.00
$\mathscr{s}=2$	6.94	0.92	0.01	$\mathscr{s}=2$	0.06	0.87	0.07
$\mathscr{s}=3$	7.09	0.81	0.02	$\mathscr{s}=3$	0.01	0.08	0.91

the process is then discretized into a 27-state Markov chain, with 9 values in each regime.

Our basic calibration of the government expenditure shocks assumes that G^c is independent of effort. It implies that the calibration of this process requires setting the levels of this variable and a standard Markov transition matrix describing its law of motion. In particular, we allow three realizations: $\mathcal{G}^c = \{G^{c1}, G^{c2}, G^{c3}\}$, with $G^{c1} > G^{c2} > G^{c3}$, and the transition matrix for G^c is pinned down by two parameters:[13]

$$\pi^{G^c} = \begin{bmatrix} \phi & \frac{2}{3}(1-\phi) & \frac{1}{3}(1-\phi) \\ 2\varpi & \phi & 1-\phi-2\varpi \\ \varpi & 1-\phi-\varpi & \phi \end{bmatrix}.$$

The parameters of the transition matrix are set to $\phi = 0.965$ and $\varpi = 0.015$. These parameters, together with the state space for the shock, are used to match several moments of current government expenditures, such as the level as well as the lower 1 and upper 99 percentiles of the G to GDP ratio, the autocorrelation of the observed government consumption, and the relative volatility of government consumption with respect to output. The resulting

[12] See the appendix for more details on the estimation and the data sources.

[13] Note that this specification of the transition matrix is motivated by the one-period-crash Markov chain of Rietz (1988).

transition matrix and government shock values of G^c are given below:

$$\pi^{G^c} = \begin{bmatrix} 0.9650 & 0.0233 & 0.0117 \\ 0.0300 & 0.9650 & 0.0050 \\ 0.0150 & 0.0200 & 0.9650 \end{bmatrix} \tag{28}$$
$$G^c \in \{0.038, 0.029, 0.025\}.$$

In our benchmark model, (policy) effort affects the probability distribution over next period's realisation of government expenditure G^c. In order to parametrize the full model, we provide more structure by assuming that, given current government liabilities G^c, there are two possible distributions of tomorrow's liabilities, $\pi^l(\cdot|G^c)$ and $\pi^h(\cdot|G^c)$, and $\pi^h(\cdot|G^c)$ first-order stochastically dominates $\pi^l(\cdot|G^c)$ for all G^c. In particular, there is $\zeta(e) \in (0,1)$, with $\zeta'(e) < 0$ and and $\zeta''(e) < 0$, such that $\pi^{G^c}(G^{c\prime}|G^c, e) = \zeta(e)\pi^l(G^{c\prime}|G^c) + (1-\zeta(e))\pi^h(G^{c\prime}|G^c)$.

To determine these two matrices, we assume that with $\bar{\zeta} = \mathbb{E}\zeta(e)$ evaluated at the ergodic distribution of effort in the IMD economy, π^h and π^l replicate the transition matrix of G^c without moral hazard in (28), subject to the requirement that π^h first order stochastically dominates π^l. There are many combination of matrices satisfying this requirement. We chose among those the matrices which allow effort to have the most effect on the probability distribution of next period government expenditures. More specifically, the matrices we use are (see the Appendix for more details on how we construct these matrices):

$$\pi^h = \begin{bmatrix} 0.93 & 0.0466 & 0.0234 \\ 0 & 0.99 & 0.01 \\ 0 & 0 & 1 \end{bmatrix}, \ \pi^l = \begin{bmatrix} 1 & 0 & 0 \\ 0.06 & 0.94 & 0 \\ 0.03 & 0.04 & 0.93 \end{bmatrix}$$

Note that the observed distribution of G^c constraints the possible effect of effort. Even in this 'extreme' case, the effect of effort is limited. For example by moving effort from 0 to 1 the borrower can increase the chance of reducing government expenditure from 0 to only 7% if the current expenditure is very high.

Last, for the $\zeta(e)$ function determining how effort decreases the weight of the *bad distribution*, we assume it to be $\zeta(e) = (e-1)^2$, which, together with the specification of $v(e) = \omega e^2$, allows us to have a simple closed form solution for effort. This functional form implies simple expressions for $\frac{\partial \pi^G(G^{c\prime}|G^c,e)}{\partial e}$ and $\frac{\partial^2 \pi^G(G^{c\prime}|G^c,e)}{\partial e^2}$ as follows:

$$\frac{\partial \pi^G(G^{c\prime}|G^c, e)}{\partial e} = -\zeta'(e)\big[\pi^h(G^{c\prime}|G^c) - \pi^l(G^{c\prime}|G^c)\big] = 2(1-e)\big[\pi^h(G^{c\prime}|G^c) - \pi^l(G^{c\prime}|G^c)\big]$$
$$\frac{\partial^2 \pi^G(G^{c\prime}|G^c, e)}{\partial e^2} = -\zeta''(e)\big[\pi^h(G^{c\prime}|G^c) - \pi^l(G^{c\prime}|G^c)\big] = -2\big[\pi^h(G^{c\prime}|G^c) - \pi^l(G^{c\prime}|G^c)\big].$$

In addition, with ω calibrated to 0.1, the optimal effort policy in equilibrium implies a value of $\bar{\zeta} = \mathbb{E}\zeta(e)$ such that $\pi^{G_c} = \bar{\zeta}\pi^l + (1-\bar{\zeta})\pi^h$, replicating the transition matrix matrix of G^c for the economy without moral hazard.

As mentioned earlier, the iid component G^d of government expenditures is introduced to improve the convergence properties of the IMD economy and it always takes a very small value. In particular, we assume that it is uniformly distributed over $[-\bar{m}, \bar{m}] = [-0.0005, 0.0005]$ and we discretize G^d into $N_d = 11$ equally spaced grid points $\{G^{di}\}_i$ over the interval, with $G^{d1} = -\bar{m}$, $G^{dN_d} = \bar{m}$, and $\Pr(G^{di}) = 1/N_d$ for all i.

4.2 The IMD Model Fit

Table 3 provides an exhaustive account of our benchmark calibration. To compute the moments we execute 2000 short run simulations of the IMD model with 300 periods each, and we discard the first 100. Further, we HP filter the simulated data to compute the second moments.[14]

The IMD economy matches most moments quite well with the notable exception of the behaviour of the average *primary surplus to GDP ratio*. In particular, the model is able to produce a significant amount of debt together with a realistic level, volatility and cross-correlation of spreads, but it has a positive average primary surplus to GDP. Note that, in any stationary model without growth, whenever there is debt in the long run, we need to have primary surplus which allows the country to pay the interest rate on its debt. This is not true in the data, as the countries in the sample were able to run deficits and increase their debt, possibly expecting growth, given that there is (moderate) growth during the sample period. What is more important than the level for our purpose, however, is that we match well the volatility of the primary surplus and our model produces a low correlation of primary surplus with GDP. Note that this correlation in the data is negative, while consumption insurance in the model requires a strong positive relationship: resources should come in whenever the country's output is low.

Finally, note that we cannot match the positive correlation of labor and GDP in the data with our current preferences. They exhibit a strong income effect that makes the country work more when is close to the borrowing limit. However, this is not the focus of our inquiry and – except for the fact that welfare comparisons are easier with separable preferences – our main results do not depend on our specific choice of preferences.

[14]Note that there is default in the IMD economy, in which case debt and the primary surplus are zero, by construction, and the spread is not defined. Therefore, all the moments involving the debt to GDP ratio, primary surplus over GDP and the spreads are conditional on borrowing (i.e. not in default).

Table 3: IMD Model Fit

1st Moments	Data	IMD
Mean		
Debt to GDP ratio	77.29%	78.6%
Real Bond Spread	3.88%	3.61%
G to GDP ratio	20.18%	19.45
Primary Surplus to GDP ratio	-0.78%	1.38%
Fraction of working hours	36.74	37.25
Average Maturity	5.38	5.38
2nd Moments		
Volatility		
$\sigma(C)/\sigma(Y)$	1.49	1.47
$\sigma(N)/\sigma(Y)$	0.92	0.70
$\sigma(G)/\sigma(Y)$	0.91	0.97
$\sigma(PS/Y)/\sigma(Y)$	0.65	0.81
$\sigma(\text{real } \textit{spread})$	1.53%	0.98%
Correlation		
$\rho(C,Y)$	0.88	0.74
$\rho(N,Y)$	0.67	-0.10
$\rho(G,Y)$	0.35	0.08
$\rho(PS/Y,Y)$	-0.29	0.13
$\rho(\text{real } \textit{spread}, Y)$	-0.35	-0.29
$\rho(G_t, G_{t-1})$	0.94	0.93

5 Contrasting the Equilibrium Allocation under the Fund and under Incomplete Markets with Default

This section compares the equilibrium allocations of an economy with the Fund and an economy with defaultable debt. We use the calibrated parameters described above for both economies. To better understand the mechanisms behind the FSF, we first present a comparison of the main moments for the two economies in Table 4, as well as the policy functions for the Fund as a function of the relative Pareto weight in Figure (1), and the policy functions for both economies as a function of debt in Figures (2)-(3). We then show representative paths of both economies, subject to the same sequence of shocks in the long run stationary distribution, in Figures (4)-(5). Finally, we study how both economies respond to a combined negative shock when they start in the long run stationary distribution: Figures (6)-(8).

5.1 Main Features of the Fund Contract

First, we compare the main statistical properties of the two allocations in Table 4. Note that, in order to obtain comparable variables (e.g. debt holdings or spreads) in the two economies, we rely heavily on Section 3. The differences between the two economies are striking.

Table 4: IMD versus Fund

1st Moments	IMD	Fund
Mean		
Debt to GDP ratio	78.6%	169.4%
Real Bond Spread	3.61%	−0.058%
G to GDP ratio	19.45%	19.21%
Primary Surplus to GDP ratio	1.38%	2.96%
Fraction of working hours	37.25	37.83
2nd Moments		
Volatility		
$\sigma(C)/\sigma(Y)$	1.47	0.36
$\sigma(N)/\sigma(Y)$	0.70	0.61
$\sigma(G)/\sigma(Y)$	0.97	0.53
$\sigma(PS/Y)/\sigma(Y)$	0.81	0.92
$\sigma(\text{real } spread)$	0.98%	0.023%
Correlation		
$\rho(C,Y)$	0.74	0.59
$\rho(N,Y)$	−0.10	0.93
$\rho(G,Y)$	0.08	0.03
$\rho(PS/Y,Y)$	0.13	0.95
$\rho(\text{real } spread, Y)$	−0.29	0.26

The Fund contract is designed to prevent a permanent level of redistribution from the fund to the borrower country. Nevertheless, the Fund is able to support much higher debt levels than the IMD economy. Given that the borrower is effectively more impatient than the lender (the markets), from the *ex-ante* perspective, this implies welfare gains. In contrast with the positive and highly volatile spreads in the IMD economy, we see very low and negative spreads in the fund that are much less volatile. Note also that the negative correlation between the spread and output in the IMD economy reflects why the sovereign debt financing does not really work. An increase in the spread in bad times, as opposed to what happens in the fund, imposes an even bigger strain on the country and hence effectively limits borrowing. In fact, looking at the much lower correlation of consumption relative to output with the fund, it becomes clear that this regime provides much more insurance and consumption smoothing. It is reflected by the highly procyclical surplus (countercyclical deficit) in the fund as opposed to the mild positive correlation in the IMD economy, implying that fiscal policy is much more countercyclical in the fund which in turns lead to stabilization of consumption. Another observation is that labor supply becomes very strongly correlated with output. This is another indication of improved efficiency as in the unconstrained optimal allocation of this economy, labor supply is solely determined by productivity under a separable utility function. Finally, the mean of government expenditure hardly changes, reflecting the fact that, on average, policy effort remains roughly constant in the two allocations. In the following subsection, we examine the policy functions in the two economies to understand more deeply how the Fund works.

5.2 Policy Functions

Figure 1 displays the policy functions for the main variables in the FSF economy as a function of the shocks and the relative Pareto weight of the borrower. In what follows, TFP shocks are labeled $\theta_i, i = 1, \ldots, 27$ where $\theta_i < \theta_{i+1}$ and G^c shocks are labeled $G_j, j = 1, ..., 3$ where $G_j > G_{j+1}$. (θ_1, G_1) is the worst combination of shocks and (θ_{27}, G_3) is the best combination of shocks. The figure displays the policies for the two extreme values of the government shock (G_1, G_3) and for relatively low and high values of the technology shock (θ_5, θ_{23}). As explained in the appendix, we simplify our computations by renormalizing the system of equations so that the policies are a function of (z, s), where $z = \frac{x}{\eta}$. With this new normalization, the law of motion for the relative Pareto weight and the optimality conditions for consumption and labor are given by:[15]

$$z'(z, s) = \eta z \frac{1 + \nu_b(z, s)}{1 + \nu_l(z, s)} + \xi(z, s) \frac{2(1 - e)\left[\pi^h(G'|G) - \pi^l(G'|G)\right]}{\pi^G(G'|G, e)} \tag{29}$$

$$c(z, s) = \eta z \frac{1 + \nu_b(z, s)}{1 + \nu_l(z, s)} \quad \text{and} \quad c(z, s)\gamma(1 - n(z, s))^{-\sigma} = \theta \alpha n(z, s)^{\alpha - 1}. \tag{30}$$

The previous conditions illustrate important features of the Fund mechanism. In an economy with no moral hazard constraints ($\xi(z, s) = 0$), the consumption of the borrower is equal to the relative future Pareto weight, $c = z'$. Both c and z' are increasing, and labor

[15] This alternative normalization has the advantage, among other things, that it eliminates the multiplier of the lender's participation constraint from the first order condition for effort. See the appendix for details.

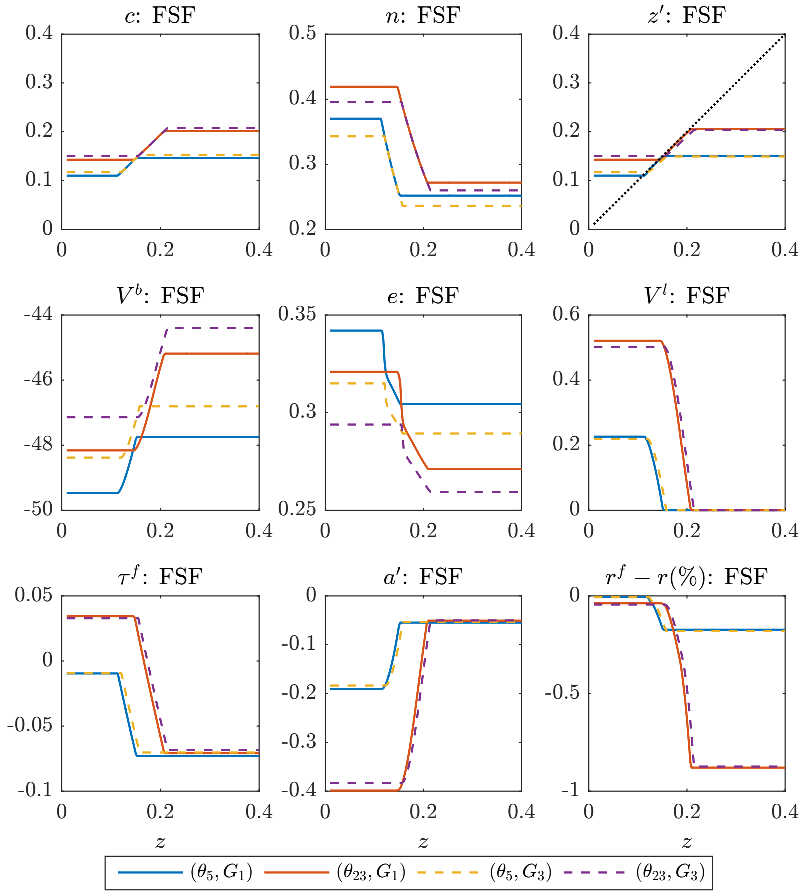

Figure 1: Optimal Fund Policies as a function of (z, s)

decreasing (due to wealth effects), in the current Pareto weight z. Moreover, in the first-best (without limited enforcement, $\nu_b = \nu_l = 0$), both consumption and the relative Pareto weight monotonically decrease over time due to the fact that the borrower is more impatient, eventually leading to his immiseration. With limited enforcement, however, such a decay, $z'(z, s) = \eta z$, is stopped by the borrower's participation constraints. In the upper left panel of Figure 1, displaying the policy function for consumption, the borrower's participation constraints define the horizontal lines on the left of the line with slope η that determines the evolution of c. As the figure illustrates, if the current Pareto weight and thus the borrower's consumption are too low, the borrower will threaten to leave the contract (his participation constraint will bind) and the planner will have to increase his relative Pareto weight to keep him from doing so. Similarly, the lender's limited enforcement constraints deter c from being too high, defining the horizontal lines to the right of the 'decay line'.

Whereas the previous qualitative features do not change for consumption and labor in the presence of moral hazard constraints, note from equation (29) that z' does depend on the future government shock. Thus, in the upper right upper panel of Figure (1) we have depicted the future Pareto weight for intermediate realizations of the states tomorrow (θ_{14}, G_2). More importantly, equation (29) shows how the Fund provides incentives. For low levels of G', we have that $\pi^h(G'|G) - \pi^l(G'|G) > 0$, providing a signal about high effort. As a a result, equation (29) implies that the borrower is rewarded with higher Pareto weight or 'better borrowing conditions'.

The second row of Figure (1) displays the borrower's value and effort as well as the value of the lender. The patterns of these policies and values can be traced back to the first row figures. The value of the borrower increases in z and mimics the Pareto weight and consumption policies, whereas the value of the lender mirrors the value of the borrower and decreases in z, since both share the surplus of the Fund. The horizontal lines on the left for the borrower's value and on the right of the lender's value reflect the respective autarky values, which is always zero for the lender due to the assumption that $Z = 0$. Similarly to labor, the effort panel reflects that the country exerts less effort as his Pareto weight increases.

In general, government expenditure shocks g play a smaller role, with the exception of effort, compared to productivity shocks θ. As a consequence of additive separable preferences, if the participation constraints are not binding, consumption is simply equal to $c = \eta z$ and it does not depend on the shocks, but it increases in productivity and decreases in the expenditure shocks when the participation constraints bind. As efficiency dictates, labor is increasing in the productivity shock and it depends only on these shocks when the participation constraints are not binding. Also not surprisingly, labor increases in the expenditure shock when the participation constraints bind, since otherwise consumption would have to drop even further. As for effort, we see that the country exerts more effort in bad productivity states and in bad expenditure shock states, in which case the country enhances the probability of moving to a good expenditure shock state to relieve the pressure on consumption.

The last row of Figure (1) displays the surplus, the asset holdings and the bond price. Recall that the surplus represents the current transfer from the country to the Fund. Increasing the Pareto weight towards the value at which the participation constraint binds for the lender, the surplus turns into a deficit and the country contemporaneously receives money from the fund, $\tau = y - c - G < 0$. The opposite is true when the Pareto weight decreases towards the value at which the participation constraints bind for the borrower. This also implies that the surplus is highly procyclical, generating countercyclical deficits and fiscal policies that enhance the insurance properties of the Fund.

Turning into the borrower's asset holdings, we have plotted a', the 'bond' component of the asset holdings that is constant across next period realizations of the shock. Here, we see a similarity with other (endogenously or exogenously) incomplete market economies, in the sense that the borrower can accumulate more debt when he has better realizations of the

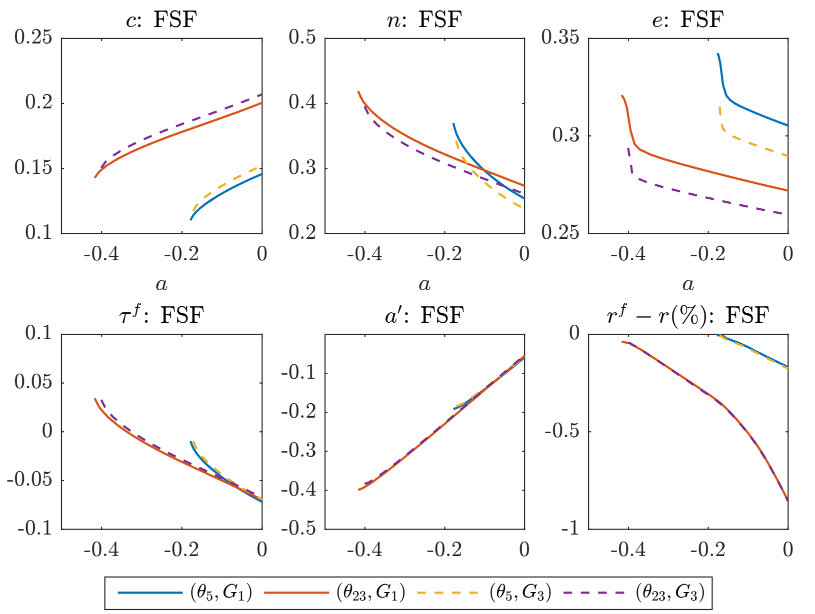

Figure 2: Optimal Fund Policies as a function of (s, a)

shocks. Given the impatience of the borrower, even if we are close to the lender's participation constraint, the country is borrowing, on average. Finally, the pattern for the spread can again be traced back to the Pareto weight. Increasing z towards the value of the lender's participation constraint will imply that in more future states the lender's participation constraint may bind. Given our discussion in section 3, this implies that the the spread becomes negative. In this region, prices discourage the country to accumulate any assets against the Fund, because that will imply that the Fund's participation constraint is binding.

We are now ready to compare the Fund and IMD policies. To do this, we have plotted the policy functions for the main variables for the incomplete markets economy with default (IMD) and for the Fund, as function of the level of debt b or a for selected values of the shocks in Figures (2) and (3) below.

As can be seen in the lower middle panel of Figure (3), displaying the level of new debt b', for a relatively bad state (θ_5, G_j), the IMD economy only allows for a very small amount of borrowing, while considerably more is borrowed in the Fund as measured by a' in the lower middle panel of Figure (2). A interesting observation, is that the level of consumption, labor supply and effort are actually quite similar in the two economies when the borrower is at his respective borrowing limit. The main difference is that, under the Fund, these limits are much looser and hence the Fund provides a much bigger buffer against shocks. We will see this more clearly in the next subsection when we compare dynamic paths of consumption and debt in the two economies.

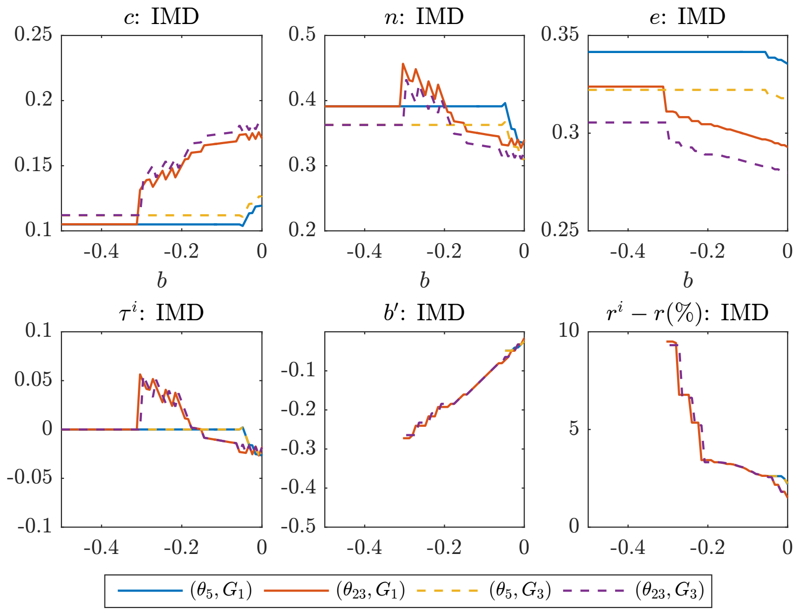

Figure 3: Optimal IMD Policies as a function of (s, b)

Looking at the surplus in the lower left panel, we also see that, in the relatively good state (θ_{23}, G_j), the IMD economy requires to run a (positive) surplus for levels of debt for which the Fund has a deficit. The spread is displayed in the lower right panel for both regimes, reflecting *positive spreads* and price collapses with default as the level of debt increases (i.e. moving to the left) in the IMD economy and, in contrast, *negative spreads* as the level of debt decreases (i.e. moving to the right) in the Fund regime.

5.3 Comparing the Economies in normal times and crisis times

It is clear from the discussion of the policy functions above that the Fund has a much larger debt-absorbing capacity and that it provides a fully state-contingent asset/payment structure. Next, we study what the impact of these differences on the time series properties of the allocations in the two economies in normal times (in the ergodic distributions) or when responding to a deep crisis.

In the first set of simulations, denoted as *Business Cycle Paths,* Figures 4 and 5 show long-run simulation initialised at the the ergodic mean of the two economies . In Figure (4), the upper left panel shows the history of shocks for 100 periods, while the output, consumption and labor allocations in the IMD and Fund regimes are shown in the other panels. In addition, Figure (5) displays the levels of effort, the surplus over GDP, debt over GDP and spreads in the two economies. In order to make the two economies comparable, we plot simulations in which they face exactly the same sequence of productivity and government expenditure

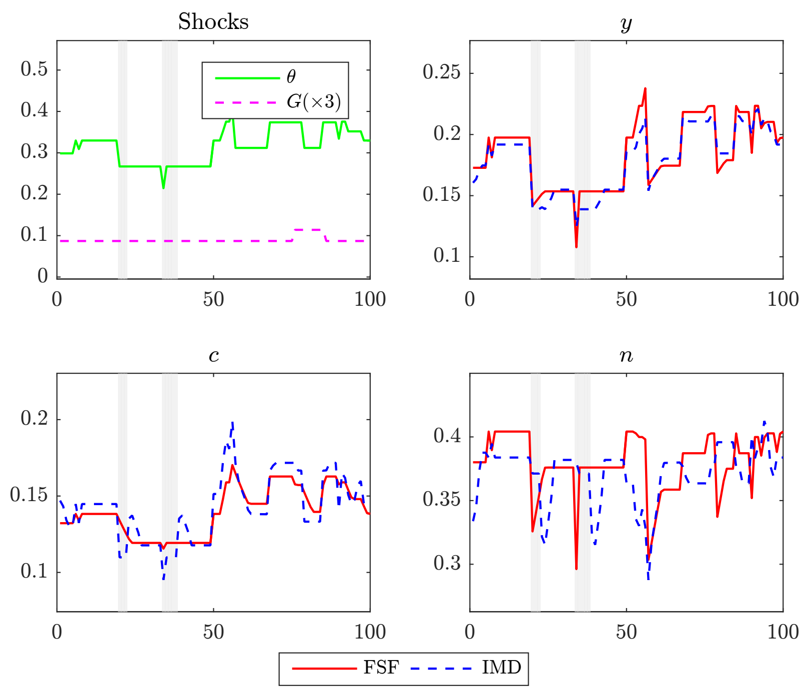

Figure 4: IMD vs. Fund Business Cycle Paths: Shocks and Allocations

shocks.

The grey periods in the figures correspond to periods of default in the IMD economy. As we see, defaults are primarily associated with drops in productivity that are accompanied by relatively large levels of debt, although in general not all drops of productivity trigger defaults. The frequency of default and the long-term nature of debt implies that spreads are high in the IMD economy, especially in periods just before a default episode, making borrowing very costly in this economy. In contrast, the Fund economy is able to accumulate a much larger stock of debt and still faces no positive spreads and occasionally (when productivity is high) has a tiny negative spread.

Figure (4) also confirms the results in table 4 regarding the lower volatility of consumption under the Fund and Figure 5 explains how this improved risk sharing is achieved. The Fund economy has a much more volatile primary surplus. More importantly, the primary surplues is countercyclical, implying that drops in productivity are balanced by increases in transfers. These episodes are also represented by (potentially large) drops in debt under the Fund as opposed to costly (with a significant loss in output and consumption) default episodes in the IMD economy. These positive transfers/debt reductions provide those insurance properties of the Fund that the IMD economy cannot replicate due to the fact that the only contingency in the financial structure is a very costly default.

As opposed to the rest of the variables, we see that effort moves much more with the

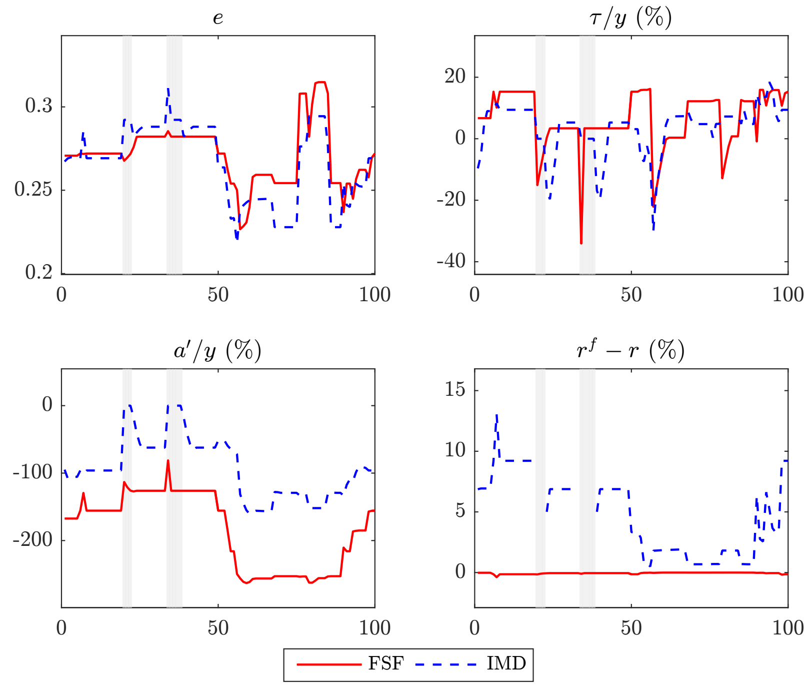

Figure 5: IMD vs. Fund Business Cycle Paths: Effort, Surplus and Financial Variables

expenditure shock than with the productivity shock, with considerably higher effort when the expenditure shocks are bad. Interestingly, this figure indicates that, on average, the Fund and the markets provide similar incentives for exerting effort. We will discus this further below when we study the crisis situation.

In the second set of simulations, denoted Impulse Responses – Figures (6) and (7) – we study the average impact of a severe negative shock. Namely, we assume that both economies hit by $(\theta, G) = (\theta_1, G_1)$ at time zero. We consider many independent economies with initial asset holdings drawn from the ergodic set of the asset distribution. After the shock at time zero, these economies draw realizations of the shocks from the (partially endogenous) Markov structure of our economy. In the figures, we report the average impulse response from 50000 independent simulations.

The smooth path of all the key variables in the previous two pictures reflects the fact that we depict the average path for many independent economies. It is important to note that there are many default episodes in the IMD economy, generating the positive spreads in Figure (7). For the real variables (shocks, output, consumption, labour, effort and primary surplus), we take an average over all economies in every period, while for debt over GDP and the spread we only average over for those who are not in default, as these variables are not defined for those who are in default.

The paths for consumption and labor clearly indicate that the Fund is able to mute the

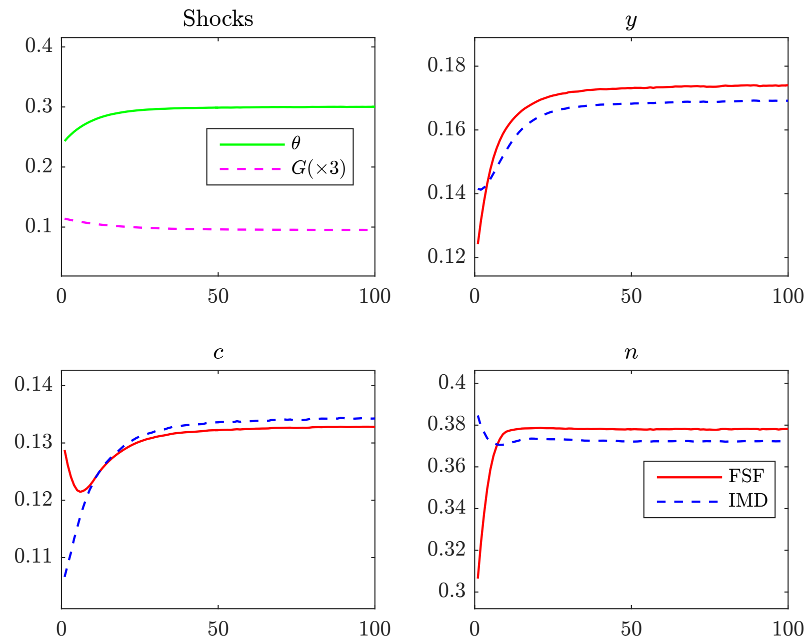

Figure 6: Shock impulse-responses: Shocks and Allocations

crisis much more, as the short run response of consumption is much smaller and, due to efficiency considerations, the Fund allows for a reduction in labor supply. At the same time, for the IMD economy, labor supply needs to increase exactly when productivity is low to limit the consumption drop. In turn, the lower labor supply implies that output drops more under the Fund. Inspecting Figure (7), we see how consumption smoothing is achieved in the Fund. First of all, under the Fund, the borrower is able to deal with a crisis by running a large deficit during the first few periods of the crisis. This is accompanied by a larger reduction of the debt. This debt reduction is due to the state contingent nature of the Fund contract: the country is (partially) insured against severe negative shocks.

It is important to note that one has to interpret the paths of debt and the spread under the IMD economy with caution, since we only depict these two variables for the selected set of countries that are not in default. In particular, the fact that debt drops under the IMD economy compared to the long run average is due to the fact that only those countries who have (significantly) lower debt than the average will not default after these severe crisis. This also explains why the spread is lower just after the crisis, these low debt countries have a low probability of default. This is confirmed by Figure (8), displaying the proportion of countries defaulting over time.

In the long run, average consumption in the fund is slightly lower, while average labor supply is slightly higher. This is is due to the fact that the country accumulates (on average)

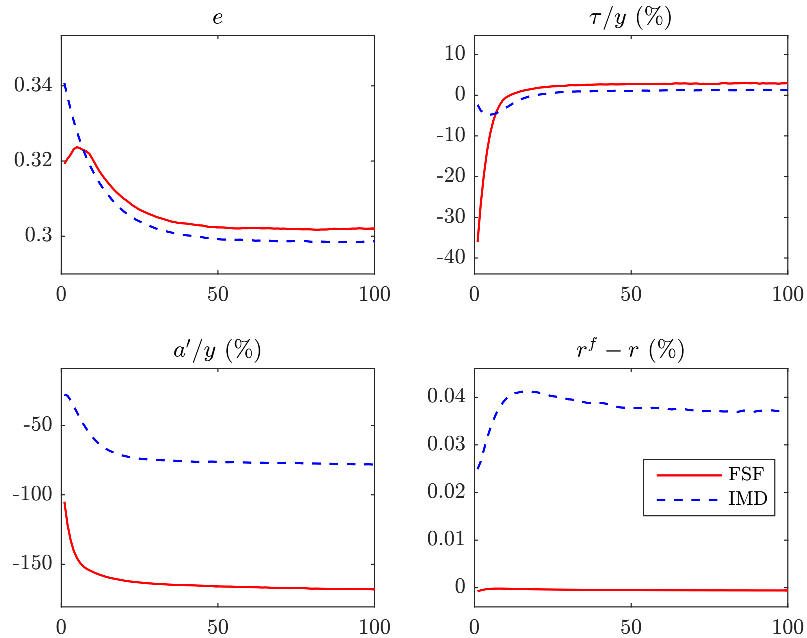

Figure 7: Shock impulse-responses: Effort, Surplus and Financial Variables

a much higher stock of debt under the Fund. At first sight, this may imply that the Fund cannot improve welfare compared to the IMD economy, because of lower long term average consumption and leisure. Note, however, that this is offset by the two other features of the Fund contract. First, as we have seen above, it offers a much smoother consumption. Second, given the fact that the borrower is more impatient than the lender, this frontloading of consumption is increasing *ex-ante* welfare. We will quantify these gains in the next subsection.

Finally, the response of effort shows an interesting pattern. Although, we have seen that, on average, the Fund and the market seem to provide the same incentives for exerting effort, the IMD economy imposes more discipline than the Fund in bad times and somewhat less in normal times. As we see, effort increases in the IMD economy more significantly than in the Fund right after the bad shock. In the long run, however, effort is slightly higher in the Fund. In the IMD economy, incentives are provided through prices and through the fact that when a country is effectively borrowing constrained higher effort increases the probability of a budget relief (a lower government expenditure). These channels are stronger in crisis times (and under temporary autarky) than in normal times. At the same time, the Fund provides incentives for exerting effort also in normal times.

This subsection has demonstrated three key properties of the Fund. First, borrowers under the Fund have access to a much larger debt capacity. Second, the state contingency of the Fund allocation leads to more risk sharing through counter-cyclical primary deficits.

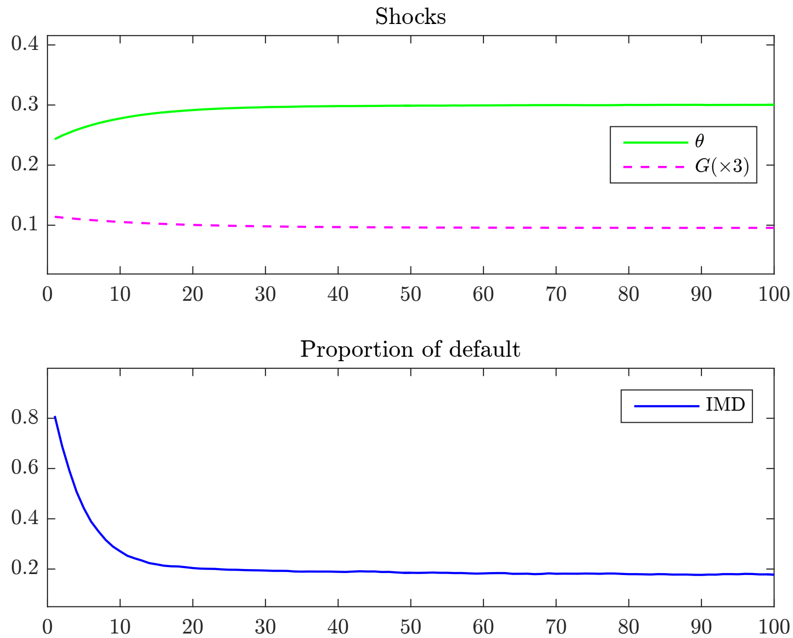

Figure 8: Shock impulse-responses: Proportion of Countries in Default

Third, costly default episodes are avoided in the Fund. In the next subsection, we compute the welfare gains associated with the Fund and we try to quantify how these different channels of welfare gains contribute to its overall desirability.

5.4 Welfare Implications and Confronting 'Debt-overhang' Problems

Table 5 shows the increased capacity to absorb debts and the welfare gains of the Fund regime compared to the IMD economy. The first column displays the welfare gains of the Fund in (annual) consumption equivalent terms when countries have zero debt for different values of the shocks (θ, G). Note that we measure the gains at zero debt because the borrowing constraint in the IMD economy is very close to zero for the worst combination of shocks (θ_l, G_h). Hence, this is the only level of debt which is comparable across regimes for all the possible shock combinations. The second and third columns of the table display the maximum end of period debt to output ratio in percentage terms that the country can have for different values of the shocks.[16] These latter measures are intended to capture the absorbing debt capacity of the borrower. Debt capacity is not straightforward to measure in the IMD economy, as there are no explicit debt limits. However, given the impatience of

[16]For this exercise, we set that the value of the idiosyncratic component of government expenditure to its zero mean.

the borrower, the actual debt choices reflect the debt capacity in this case. Hence, we choose the highest debt/output ratio for a given state s across all feasible levels of current debt b. In the case of complete markets, the borrower has a whole portfolio of debt (and assets) for each future state. However, in section 3.3, we have shown that from any portfolio of Arrow securities, we can construct a bond component a' and an insurance portfolio, with the bond component being the comparable measure to the debt choice in the incomplete markets economy. Given this, we follow the same logic and we present the maximum of debt/output ratio using the bond component across all values of current debt for a given state.[17]

The difference between the debt capacities in the two economies are striking. As we see, the Fund is able to absorb much higher debt-to-GDP ratios in all the states, while the capacity to absorb debts in the IMD economy is substantially smaller, particularly in bad states. Given the relatively high persistence of the shocks, a low realization of the shock today implies a high spread on any significant amount of debt as the country will have only a small chance to pay it back through a better realisation of the shock. Moreover, since, due to the asymmetric default penalty specification, there is no output penalty for low shock realizations, default is not particularly costly in this case (with high probability). Another interesting feature of the IMD economy is that the borrowing limits are relatively loose in normal times (for medium productivity levels). This is due to the fact that, in this case, the countries suffer an output loss upon default and the value of staying in the financial markets is higher in relative terms. Nevertheless, the Fund will be able to support much more borrowing. As we will see below, given the relative impatience of the borrower ($\beta(1+r) < 1$), this increase in debt capacity is very valuable for the borrower.

The table also reflects that welfare gains are very substantial with the Fund: the consumption-equivalent steady-state average welfare gain is around 5 percent and, even more relevant, the gain is of 5.91 percent in the worst state. As discussed earlier, two of the features of the Fund that lead to welfare gains are the fact that it provides more risk sharing through state contingent assets and the fact that it allows for a much higher debt capacity, both particularly important with bad shocks. In other words, the welfare gains of the Fund are the highest when the country is in trouble, although the gains are still substantial when the country is hit by good shocks. Note that this is partially due to the fact that agents are forward-looking and gain benefits from the future insurance against bad shocks, and partially because at the higher shock levels they still have a much higher debt capacity and still benefit from the state contingency of the Fund contract.

[17] Note that, for the Fund economy, one can actually compute the maximum borrowing capacity as the bond component of the portfolio that allows for the maximum amount of borrowing across all possible realizations of the future shocks: $a' = \frac{\sum_{s'|s} q(s'|s) A_b(s')}{\sum_{s'|s} q(s'|s)}$, where $A_b(s')$ is the state contingent borrowing constraint of the country. However, for comparison with the incomplete markets economy, we choose the alternative measure described in the text, which has limits that are necessarily tighter than the maximum borrowing limit.

Table 5: Welfare comparison at zero debt

(θ, G)	Welfare Gain %	$\max \frac{-b'(s,\cdot)}{y(s,\cdot)}$ %	$\max \frac{-a'(s,\cdot)}{y(s,\cdot)}$ %
$(\theta_l, G_h) = (0.148, 0.038)$	5.91	1.71	66.16
$(\theta_m, G_h) = (0.299, 0.038)$	5.59	107.61	165.08
$(\theta_h, G_h) = (0.456, 0.038)$	3.76	215.15	317.09
$(\theta_l, G_l) = (0.148, 0.025)$	5.07	1.84	67.12
$(\theta_m, G_l) = (0.299, 0.025)$	5.14	111.47	164.63
$(\theta_h, G_l) = (0.456, 0.025)$	3.55	214.78	313.82
average	5.04		

Next, we go deeper to inspect how important are these different features of the Fund for the welfare gains. To do this, we propose a novel decomposition of welfare gains that implements a series of counterfactual exercises to evaluate the main channels of welfare improvements identified in the previous subsection. The first important difference between the IMD and Fund economies is that default occurs in equilibrium in the IMD economy but not in the Fund economy. Given this, we first simulate a counterfactual IMD economy where we keep the asset prices and asset decisions at the same level, while default and return to the market happens under exactly the same circumstances as in the benchmark IMD economy, except that no output penalty is imposed. When we compare the value of this counterfactual economy with the value functions of the IMD economy, we obtain the isolated effect of the output penalty. Next, we isolate the second penalty of default from the output penalty, namely, market exclusion. We modify the previous counterfactual economy by allowing the countries to always come back to the market after one period of default. Comparing the value in this case with the value of the previous counterfactual gives us the isolated effect of market exclusion. Third to evaluate the effect of a higher debt capacity, we solve counterfactual economies with looser exogenous debt limits and no default. In particular, the debt limits are set at the endogenous borrowing constraints associated with different values of the state vector s under the Fund economy. Comparing the value of this counterfactual exercise to the previous one provides us with the measure of welfare gains due to an increased debt capacity in the Fund. Finally, note that the previous three counterfactuals do not account for the fact that Fund is able to provide state-contingent payments as opposed to the IMD economy (apart from the costly default episodes). This is captured by the (residual) difference between the welfare in the Fund economy and the third counterfactual (see the Appendix for more details). The results of the counterfactuals are displayed in Table 6 below for a selection of initial states:[18]

[18] For each row of the Table, we have imposed the endogenous debt limit of the Fund corresponding to

Table 6: Welfare Decomposition

(θ, G)	Penalty %	Exclusion %	Debt Capacity %	State Contingency %
(θ_l, G_h)	4.21	0.76	42.58	52.44
(θ_m, G_h)	16.98	4.22	56.77	22.03
(θ_l, G_l)	4.76	1.05	40.60	53.59
(θ_m, G_l)	18.78	4.37	49.56	27.29

The table displays the percentage contribution of each of the four factors mentioned above to the overall welfare gain. The table reflects that, for all values of the shocks, the higher debt capacity and insurance through the state contingent assets provided by the Fund are the two most important factors contributing to the welfare gains. In particular, for the worst combination of shocks (θ_l, G_h), these two factors account for 95% of the welfare gains, while they still account for 77% of the gains with a medium productivity shock and a good expenditure shock (θ_m, G_l). We also see that the contribution of not having a penalty upon default is very small for low productivity shocks but is non trivial for medium productivity shocks. The reason is that our calibrated default penalty function do not impose penalties for low productivity levels only for the medium and higher productivity levels. This implies that the penalty will matter more with a medium productivity shock, not only because the penalty is directly imposed in that state, but also because if the shock is higher there is a higher chance of moving to a higher state and incurring a higher penalty in the future. In general, we also see that the fact that a country is excluded from the financial markets upon default does not see to be very important, probably because exclusion is only temporary. The key result is that both the increase in debt capacity and the state contingency of payments are quantitatively relevant in explaining the welfare gains. For low level of the initial state or countries in crisis, the state contingency (insurance) provided by the Fund contract is the most relevant factor but the increased debt capacity has a similar importance. The increased debt capacity becomes quantitatively more relevant when we increase the debt capacity even further and we consider as an initial state an intermediate level of productivity. This is due to two reasons. First, these countries have a higher level of consumption and hence, in relative terms, they may appreciate more the better intertemporal allocation of consumption that is

the given initial state as an exogenous fixed borrowing limit for all shocks in the counterfactual of column 3. This implies that, when we consider the limit associated with low levels of productivity, the limits are much tighter than under the Fund for medium or high levels of future productivity shocks. Similarly, when we consider medium productivity levels as the initial state (the second and fourth rows), the limits we impose are even looser than those under the Fund for low levels of future productivity. Due to this latter issue, we have restricted the welfare decomposition to only low and medium productivity levels.

allowed by more debt capacity than the state-contingency of the contract. Second, the looser debt limits allow for potentially even more consumption smoothing than in the original Fund economy for low levels of shocks because these limits are even looser than the endogenous limits under the Fund for these shocks.

To summarize, the Fund leads to substantial welfare gains that arise primarily from the fact that it provides insurance through the state contingent assets as well as a higher debt capacity, while it avoids costly default episodes that impose direct penalties on productivity.

6 Conclusions

By developing and computing a model of a *Financial Stability Fund* as a *constrained-efficient mechanism* we have contributed to the existing literature on risk sharing and sovereign debt, and to the current policy debate on risk-sharing and shock-absorbing mechanisms for the European Economic Monetary Union (EMU). In particular, we have quantitatively shown that the visible welfare gains of a well designed Fund can be substantial, even if we have calibrated the model for euro area 'stressed countries', and we have set a 'tight constraint' on risk-sharing transfers: the fund should always have non-zero expected profits from its Fund contracts. We have also shown that accounting for *moral hazard* does not substantially change the Fund allocations, although these (incentive compatibility) constraints interact with limited enforcement constraints, distorting effort and making *negative spreads* more likely to emerge. In our economies, the moral hazard problem only affects the distribution of government expenditures. If, however, it were to affect productivity shocks too (e.g. through costly structural reforms) the effect may be greater – an issue that we leave for future research.

Similarly, we leave for future work to study, and quantify, how the Fund can be more effectively simplified – in the sense of making it less contingent, or relying on a simpler financial structure – as has been proposed (e.g. a 'rainy day' Fund to absorb 'large economic shocks'). The advantage of our framework is that it allows for a characterizations and quantitive evaluation of the tradeoff between *simplicity* and *efficiency*, providing a guide for further 'contractual engineering' work which should help its implementation.

While the Fund has been designed as a *risk-sharing mechanism*, we have shown it is also an effective and 'robust crisis management' mechanism. Furthermore, Fund contracts help to *stabilize* the economy by generating and enhancing counter-cyclical fiscal policies. The Fund can also be used to address sovereign 'debt-overhang'problems, since it has high absorbtion capacity; in particular, it is the self-enforcing stabilisation and default-free nature of the Fund that gives it its credibility and its capacity to absorb large existing debts – or provide generous credit in times of crisis – in contrast to existing debt market instruments. Existing crisis-resolution institutions – such as the ESM – are able to absorb relatively large debts, but by relying on *ex-ante* conditionality, instead of relying on *ex-post* conditionality, as in our *constrained-efficient mechanisms*, they do not exploit all the potential welfare gains of having long-term contracts.[19] Our work may be useful to them. Finally, a central feature of the Fund is its capacity to transform risky debt contracts into safe Fund contracts, which become safe assets in the balance sheet of the Fund, against which it can issue safe bonds.

Appendix

A Constrained efficiency of the RCE

In Section 3.3, we have shown that a *constrained-efficient Fund contract* can be decentalized as a *recursive competitive equilibrium.* Although it is not the focus of our analysis, two remarks are in order regarding whether the reverse is also true – i.e. whether a RCE implements Fund allocations and, therefore, it is constrained-efficient. The argument is to revert the steps in the derivation of the main text – i.e. to go from the asset prices and optimal policies of RCE to the optimal policies and the structure of the Fund contract – to show that, provided that a RCE exists, there is a corresponding *constrained-efficient Fund contract.* This argument – based on Alvarez and Jermann (2000) – is valid if there is no effort decision – i.e. $e(a, s)$ is exogenously given – but is not when, as in the economies under study, the borrower decides the level of effort.

When effort is exogenous the RCE is a recursive version of an Arrow-Debreu equilibrium with limited enforcement constraints. The corresponding identity between asset prices and maximal kernels, the first order conditions of agents' problems in the RCE, the fact that present value budgets are satisfied and, limited enforcement constraints satisfied, provide the basis to show that the RCE is constrained-efficient and – using the identity between the evolution of the ratio of marginal utilities of income and (the inverse) of the weights in the recursive planner's problem x – one can derive the Fund contract.

However, while this is a remarkable, but not novel, result, it is of limited practical value. Is not obvious how the market can assess the endogenous limited enforcement constraints. As we have seen, in our context, interest rates only change when the lender's limited enforcement constraint is binding, there is no price information about the borrower's intertemporal participation constraints, yet these are always accounted for.[20] This is not a trivial problem for the Fund either, but at least it is part of the design of the Fund contract. In other words, as above, asset prices are the result of this design, not the result of competitive forces at work. While this first remark applies more broadly to the literature on price-decentralization of contracts, the second is specific to our economies where effort, affecting underlying uncertainty,

[19]For example, as of May 2017, the ESM is holding 49.4% of Greece's sovereign debt (which amounts to 88.5% of Greece GDP) as long-term, over 30 years, unconditional debt.

[20]In the language of Alvarez and Jermann (2000), these solvency constraints are 'not too tight', they just avoid default when the country is indifferent on wether defaulting or not.

is an agent's choice.

The effort decisions in the Fund and in the RCE are determined by equations (3) and (21), therefore they are same if the value functions are the same, but they are not if we start from a RCE. As we have seen in solving for the Fund's effort – given by (9) – there is an externality, a social value of effort, which in principle the borrower does not account for. Nevertheless, the Fund takes as a restriction the equality between 'non-accounted' marginal costs and benefits – in (10). In other words, the value function in (3) already endogenizes the social value of effort, which is not the case in (21). We have circumvented this problem, while decentralizing the Fund contract, by postulating that both value functions were the same *and*, instead of using the *endogenous* borrowing constraint of the RCE (20) and (23), imposing the limit asset holdings of the Fund – in (26) and (27), – validating this way the assumption that both value functions were the same. In other words, in decentralizing the Fund, the RCE value function is derived directly from the Fund value function and, therefore, the endogenous borrowing constraints, as well as the welfare properties, are given by the Fund. But to show that the the effort policy of RCE is (constrained) efficient and then get the effort policy of the Fund we should keep the *endogenous* borrowing constraints (20) and (23), and proceed differently.

A.1 The RCE with Pigou taxes

As we have seen, with *moral hazard* the borrower's effort decision (21) must endogenize the 'non-accounted' social effect (10). This can be done by properly changing the borrower's value, $W^b(a'(s'), s')$, on the right-hand side of (21), with Pigou taxes, which then must be incorporated in the definition of a RCE. Let $\tau^e(s)$ be a lump-sum tax (conditional on the state) and $\tau^r(s|\ s_-)$ a lump-sum subsidy – i.e. a positive or negative reward – in state s conditional on the state the previous period being s_-. These taxes and subsidies satisfy $\tau^r(s_0|\ s_{-1}) = 0$ and the following non-arbitrage condition: $\tau^e(s) = \sum_{s'|s} Q(s'|s)\tau^r(s'|s)$.

A *recursive competitive equilibrium with Pigou taxes* is a RCE, where the intertemporal budget constraint of the borrower is:

$$c + \sum_{s'|s} q(s'|s) \left(a(s') - \delta a(s)\right) + \tau^e(s) \leq \theta(s)f(n) - G(s) + (1 - \delta + \delta\kappa)\, a(s) + \tau^r(s|\ s_-),$$

and, correspondingly, the budget of the lender is:

$$c_l + \sum_{s'|s} q(s'|s) \left(a_l(s') - \delta a_l(s)\right) + \tau^r(s|\ s_-) \leq (1 - \delta + \delta\kappa)\, a_l(s) + \tau^e(s).$$

Note that, being lump-sum, the Pigou taxes do not change the first-order conditions of the agents, nor their present value budgets in period zero. However, they can change the endogenous borrowing constraints on asset holdings, given by (20) and (23). In particular, equations (24) and (25) now become:

$$\begin{aligned} a_b(s^t) &= \sum_{n=0}^{\infty} \sum_{s^{t+j}|s^t} Q(s^{t+j}|s^t)\left[c(s^{t+j}) + \tau^e(s^{t+j}) - \left(\theta(s^{t+j})f(n(s^{t+j})) - G(s^{t+j})\right.\right. \\ &\qquad\qquad\qquad\qquad\qquad \left.\left. + \tau^r(s^{t+j}|s^{t+j-1})\right)\right] \\ &= -\sum_{n=0}^{\infty} \sum_{s^{t+j}|s^t} Q(s^{t+j}|s^t)c_l(s^{t+j}) + \tau^r(s^t|s^{t-1}) \\ a_l(s^t) &= \sum_{n=0}^{\infty} \sum_{s^{t+j}|s^t} Q(s^{t+j}|s^t)c_l(s^{t+j}) - \tau^r(s^t|s^{t-1}), \end{aligned}$$

where c, n and c_l are competitive choices of the borrower and the lender, respectively. The endogenous borrowing constraints (20) and (23) can also be expressed as: $W^b(a_b(s^t), s^t) \geq V^a(s^t)$ and $W^l(a_l(s^t), s^t) \geq Z, \ t > 0$.

Nevertheless, it is the social optimal choice of effort – e.g. given by the Fund adding equation (10) to (21) – what defines the Pigou taxes as follows:

$$\tau^e(s) \equiv \text{NMC}(s) = \tilde{\xi}(x,s)v''(e(x,s))$$

and

$$\begin{aligned} \tau^r(s'|s) &\equiv \frac{\text{NMB}(s'|s)}{\max\left\{\frac{u'(c(s'))\eta}{u'(c(s))}, 1\right\}} \\ &= \left[\tilde{\xi}(x,s)\eta \frac{\partial^2 \pi^G(G'|G, e(x,s))/\partial e \partial e}{\pi^G(G'|G, e(x,s))} V^{bf}(x', s')\right. \\ &\qquad \left. + \frac{1+\nu_l(x,s)}{x} \frac{\partial \pi^G(G'|G, e(x,s))/\partial e}{\pi^G(G'|G, e(x,s))} V^{lf}(x', s')\right] \\ &\qquad \times \left[\max\left\{\frac{1+\nu_l(x',s')}{1+\nu_b(x',s')} \frac{1}{1+\frac{\varphi(s'|x,s)}{1+\nu_b(x,s)}}, 1\right\}\right]^{-1}. \end{aligned}$$

In sum, in a *Recursive Competitive Equilibrium with Pigou taxes* the market (or the government) must figure out these taxes too.

B Data Sources and Measurement

The primary data source we use is the AMECO dataset. We use annual data for the 5 Euro Area 'stressed' countries, and except for a few series, the sample coverage is 1980–2015. Table 7 provides a summary of the data sources and definitions. We construct model consistent measures based on the raw data. In what follows, we provide details on the sources and measurement methods.

Table 7: Data sources and definitions

Series	Time periods	Sources	Unit
Output	1980–2015	AMECO (OVGD)[a]	1 billion 2010 constant euro
Private consumption	1980–2015	AMECO (OCPH)	1 billion 2010 constant euro
Government consump.	1980–2015	AMECO (OCTG)	1 billion 2010 constant euro
Total working hours	1980–2015	AMECO (NLHT)[b]	1 million hours
Employment	1980–2015	AMECO (NETD)	1000 persons
Government debt	1980–2015	AMECO EDP[c]	end-of-year percentage of GDP
Primary surplus	1980–2015	AMECO (UBLGIE)[d]	end-of-year percentage of GDP
Bond yields	1980–2015	AMECO (ILN)[e]	percentage, nominal
Inflation rate	1980–2015	AMECO (PVGD)	percentage, GDP deflator
Debt maturity	1990–2010	OECD[f]	years
Labor share	1980–2015	AMECO[g]	percentage

[a] Strings in parentheses indicate AMECO labels of data series.

[b] PWT 8.1 values for Greece in 1980–1982.

[c] General government consolidated gross debt; ESA 2010 and former definition, linked series.

[d] AMECO linked series for 1995–2015; European Commission *General Government Data* (GDD 2002) for 1980–1995.

[e] A few missing values for Greece and Portugal replaced by Eurostat long-term government bond yields.

[f] Differing time coverage across countries; see the text for details.

[g] Calculated based on various series on labor compensation; see the text for details.

B.1 National accounts variables

For the aggregate output Y_{it} and government consumption expenditure G_{it} of each country, we use directly the corresponding data series from AMECO over 1980–2015, measured in constant prices of 2010 euros. Since there is no capital accumulation in the model, we interpret consumption in the model as private absorption, and define the model consistent measure in the data as the sum of the private consumption and gross capital formation. For the aggregate labor input n_{it}, we use two series from AMECO, the aggregate working hours H_{it} and the total employment E_{it} of each country over the period 1980–2015. We calculate the normalized labor input as $n_{it} = H_{it}/(E_{it} \times 5200)$, assuming 100 hours of allocatable time per worker per week. However, for most of the data moment computations, we use H_{it} directly, since the per worker annual working hours do not show a significant cyclical pattern and both the level and the trend do not affect the computation of the moments.

B.2 Government debt variables

We use the end-of-year government debt to GDP ratios in AMECO to measure the indebtedness of the Euro Area ‘stressed’ countries. The government debt is defined as the general government consolidated gross debt. This is conceptually different from the debt in the model, which corresponds to national debt more closely. Nevertheless, we use the gross debt measure, as it provides a consistent measure across countries and is arguably an upper limit on the indebtedness of the government.

We use the nominal long-term bond yields in AMECO to measure the nominal borrowing costs of the Euro Area 'stressed' countries, and use short-term interest rates in German to measure the funding cost of international investors. The risk-free rate is measured as the real short-term interest rate of Germany, which equals to the average of the nominal rate minus GDP deflator from 1980–2015. To arrive at a meaningful measure of the *real* spread, i.e., a spread unaffected by expected inflation hence rightly reflecting the 'stressed' countries' credit risk, we split the sample into to two parts divided by the period the euro was introduced. For the first part, 1980–1998, we use spot and forward exchange rates to convert the German nominal risk free rate into each stressed country's local currency, hence deriving a synthetic local currency risk free rate, and then take the difference between the local nominal long-term bond yield with the synthetic risk free rate. Since the synthetic risk free rate is denominated in the local currency as well, it is subject to the same inflation expectations as the long-term bond yield, and consequently, the difference is equivalent to the real spread. For the second part, 1998–2015, we can directly use the spread between the 'stressed' countries' long-term bond yields and the German short-term interest rate, since all rates are denominated in euro and are thus subject to the same inflation expectation.

The information on the maturity structure of the government debt for the Euro Area 'stressed' countries is not comprehensive. We were able to find average years to maturity for the five countries from 3 sources. The overall time coverage is unequal across countries: 1998–2010 and 2014–2015 for Ireland, 1998–2015 for Greece, 1991–2015 for Spain, 1990–2015 for Italy, and 1995–2015 for Portugal.

B.3 Fiscal positions

Recall that the primary surplus is defined as government surplus minus interest payments. Alternatively, by the government's budget constraint, the primary surplus can be expressed as the net lending by the government, i.e., the difference between revenue of newly issued debt and payments on interests and retiring debt. For the economy with incomplete markets and default we are considering, this equals to $q_t(b_{t-1} - b_t) - (1 - \delta + \delta\kappa)b_t$, and by the economy's budget constraint, the last expression is just equal to $y_t - c_t - G_t$, which is the measure we use for primary surplus in the model.

To be consistent with the model, we also measure the primary surplus in the data according to the last expression. Since c_t is already measured as the private absorption, i.e., sum of the private consumption and gross capital formation, the empirical measure of the primary surplus is equivalent to the net export by the national accounting identity.

B.4 Labor share

We use various data series from AMECO to construct the labor share of annual output for each of the Euro Area 'stressed' countries over the period 1980–2015. First, we use nominal compensation to employees of the total economy in AMECO (labeled by UWCD) to measure the labor income for employees. Second, to measure the labor income for self-employed people, we take the difference between two AMECO series, UOGD and UQGD, where the former is gross operating surplus and the latter is the same measure net off imputed compensation for the self-employed population. We define the total labor income as the sum of the labor income for employees and self-employed, i.e., UWCD + UOGD − UQGD. Finally, the labor share is calculated as the ratio of labor income to nominal GDP.

B.5 Labor productivity

Given the production function, $y = \theta n^{\alpha}$, we measure the labor productivity of country i at time t according to $\theta_{it} = Y_{it}/H_{it}^{\alpha}$, or equivalently, $\log \theta_{it} = \log y_{it} - \alpha \log n_{it}$. Note that we use a common α for all ' Euro Area 'stressed' countries to estimate the productivity process for each country using the individual output and labor input data. Let $\log \hat{\theta}_{it}^{o}$, denote the measured level for logged labor productivity. To compute the data moments involving the labor productivity, we use the HP-filter to detrend the sample productivity $\{\log \hat{\theta}_{it}^{o}\}$. Moreover, as we explain in the next section, we use a Markov regime switching model to estimate the parameters of the labor productivity process. Before taking the data to the model, we adjust the original sample in the following two steps:

1. We take out a common linear time trend in the $\{\log \hat{\theta}_{it}^{o}\}$ series.
2. After detrending, we further standardize $\{\log \hat{\theta}_{it}^{o}\}$ for each i so that the resulting series has the same sample mean and volatility over i. This is to prevent the level and volatility differences in $\{\log \hat{\theta}_{it}^{o}\}$ across i to induce spurious regime switching behavior in the estimation process.

We denote the adjusted sample productivity by $\{\log \hat{\theta}_{it}\}$, which is then used in the estimation of the MRS model discussed in what follows. The adjusted country specific series is illustrated in figure 9.

C Estimation of the Labor Productivity Process

Let $\{\log \hat{\theta}_{it} : i = 1, \ldots, 5, t = 1980, \ldots, 2015\}$ denote the logarithm of the measured labor productivity series of the PIIGS countries. We fit these observations to a panel Markov regime switching (MRS) model as follows:

$$\log \theta_{it} = (1 - \rho(s_{it}))\mu(s_{it}) + \rho(s_{it}) \log \theta_{it} + \sigma(s_{it})\varepsilon_{it},$$

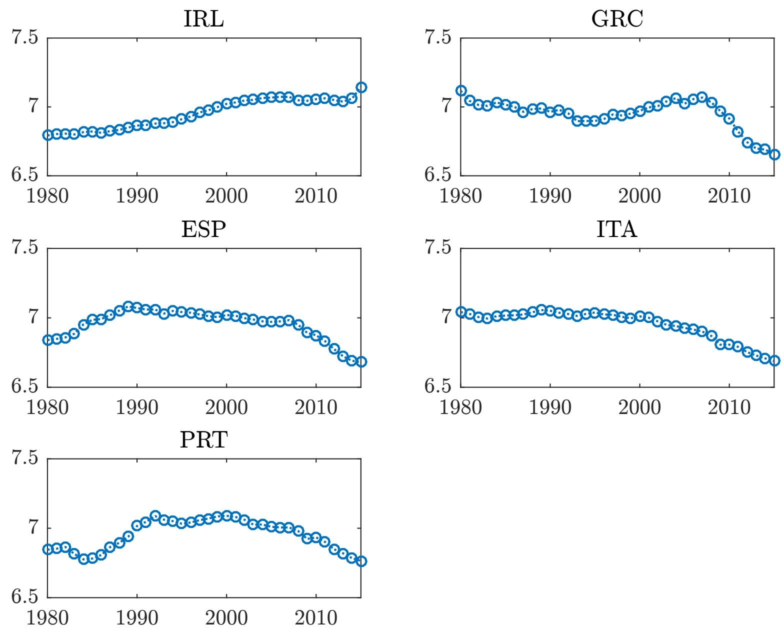

Figure 9: Measured productivity series for each country

where $s_{it} \in \{1, \ldots, S\}$ denote the regime of country i at time t, $\nu(s)$, $\rho(s)$, and $\sigma(s)$ are functions of the regime, and $\varepsilon_{it} \overset{\text{iid}}{\sim} N(0, 1)$. Country specific regime s_{it} is independent in the cross-section, and follows a Markov chain over time, with an $S \times S$ regime transition matrix π^{s}. The model is an extension of Hamilton (1989) to the panel data setup. To estimate the model, we adapt the expectations maximization (EM) algorithm outlined in Hamilton (1990) to our setup, combined with the more efficient procedure of Hamilton (1994) to calculate the smoothed probabilities of latent regimes; see Liu (2015) for the details on the estimation algorithm.

We set $S = 3$ for the panel MRS model in our estimation. Because the likelihood function of the model is highly nonlinear, the EM algorithm of likelihood maximization may be stuck at an local maximum. To overcome this potential deficiency, we randomize the initial point in the parameter space for 1,000 times. Table 8 displays a summary of the estimates, and figure 10 shows the smoothed regime probability for each country from 1980 to 2015.

We discretize the MRS log productivity process using the method of Liu (2016). The method exactly replicates regime-conditional mean, variance and autocorrelation of the original MRS process by a discrete Markov chain. While it can also replicate the unconditional mean exactly, the replication of the unconditional variance and unconditional autocorrelation is less then perfect. Nonetheless, the method allows to adjust the state space so that the discrete chain also replicates the coefficient of variation of the sample observations. In partic-

Table 8: Estimates for the productivity process

	Regime		
Parameter	1	2	3
μ	6.3473	6.9438	7.0905
ρ	0.9282	0.9212	0.8076
σ	0.0208	0.0127	0.0204
Transition Matrix			
Regime 1	0.8971	0.1029	0.0000
Regime 2	0.0653	0.8666	0.0681
Regime 3	0.0174	0.0763	0.9063
Stationary Distribution	0.3049	0.4025	0.2926

ular, we discretize the log MRS productivity process with 9 grid points for each regime, which results in a 27-state Markov chain. Table 9 summarizes the performance of the discretization.

Table 9: Comparison of Unconditional Moments from the Discretization

Parameter	Mean	Coefficient of Variation	Autocorrelation
Sample Productivity	6.95	0.11	0.97
Estimated MRS Process	6.85	0.21	0.99
Discretized Process	6.85	0.21	0.94

Before feeding the shock into the program for solving the model, we further adjust the discretized log productivity process. Specifically, we adjust the state space and leave the transition matrix untouched. First, we take exponential of the 27 states, so that we have productivity levels instead of logs. Second, we normalize the productivity level by dividing each state by a constant. The resulting productivity process $\tilde{\theta}$ has the following state space:

D Specification of the Government Expenditure Transition Matrices

First, we show that it is sufficient that the matrices $\pi^h(\cdot|G^c)$ and $\pi^l(\cdot|G^c)$ defined in the calibration section satisfy the MRL property for all G^c for $\pi^{G^c}(G^{c\prime}|G^c, e)$ to satisfy both the MRL and the CDF properties. Fixing G^c and denoting $\pi^x(G_i^{c\prime}|G^c)$ by π_i^x for $x = h, l$, the

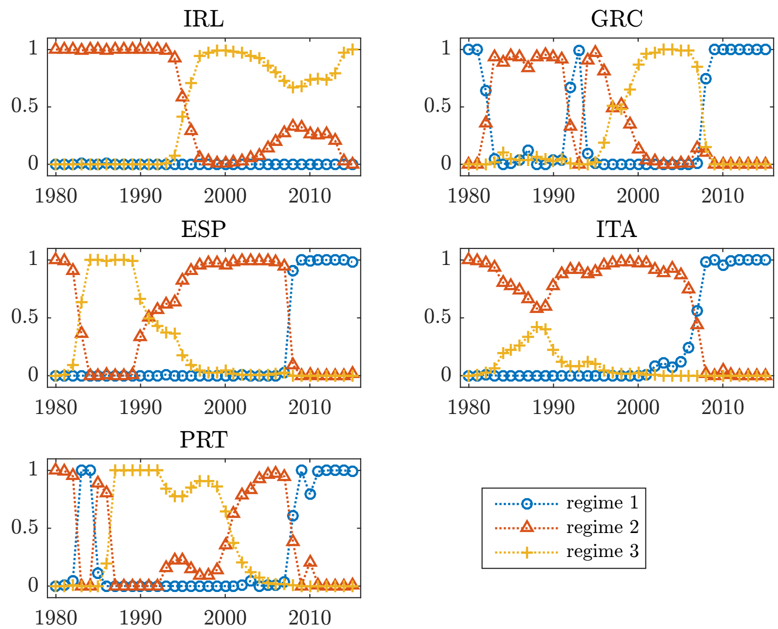

Figure 10: Smoothed regime probabilities for each country

Table 10: State Space of the Productivity Process

Regime 1:	0.1478	0.1674	0.1896	0.2147	0.2431	0.2754	0.3119	0.3532	0.4000
Regime 2:	0.1909	0.2135	0.2388	0.2671	0.2987	0.3341	0.3737	0.4180	0.4675
Regime 3:	0.2715	0.2897	0.3091	0.3298	0.3519	0.3754	0.4006	0.4274	0.4560

monotone likelihood ratio property for π^h and π^l boils down to:

$$\frac{\pi_i^l}{\pi_i^h} \geq \frac{\pi_j^l}{\pi_j^h}, \quad \text{for all } i < j,$$

with the convention that

$$\frac{\pi_i^l}{\pi_i^h} = \begin{cases} \infty, & \pi_i^l > 0 = \pi_i^h, \\ 1, & \pi_i^l = 0 = \pi_i^h. \end{cases}$$

In other words, the likelihood ratio π_i^l/π_i^h is non-increasing in i so that a higher G^c indicates higher effort. Given the monotone likelihood ratio property of π^l and π^h, it follows that π^h dominates π^l by first order stochastic dominance (FOSD). Let $F^x(\cdot)$ and $F^{G^c}(\cdot)$ denote the functions associated with $\pi^x(\cdot|G^c)$ for $x = h, l$ and $\pi^{G^c}(\cdot|G^c, e)$ that are defined in Section 4. First order stochastic dominance implies that $F^h < F^l$, and given the expression of $\partial^2 \pi^{G^c}/\partial e^2$, it follows that:

$$\frac{\partial^2 F^{G^c}}{\partial e^2} = -\zeta''(e)(F^h - F^l),$$

which is positive as long as $\zeta(e)$ is convex. As a result, $\pi^{G^c}(\cdot|G^c, e)$ satisfies the CDF property. To show that $\pi^{G^c}(\cdot|G^c, e)$ also satisfies MLR property in e, we only need to verify that

$$\frac{\partial \pi^{G^c}(G^{c\prime}|G^c, e)/\partial e}{\pi^{G^c}(G^{c\prime}|G^c, e)} = -\zeta'(e)\frac{\pi^h(G^{c\prime}|G^c) - \pi^l(G^{c\prime}|G^c)}{(1-\zeta(e))\pi^h(G^{c\prime}|G^c) + \zeta(e)\pi^l(G^{c\prime}|G^c)}$$

is non-decreasing in $G^{c\prime}$. Let e and G^c be fixed, so that $\zeta \equiv \zeta(e)$ and $-\zeta'(e)$ are positive constants. We only need to verify that

$$\frac{\pi_i^h - \pi_i^l}{(1-\zeta)\pi_i^h + \zeta\pi_i^l} \leq \frac{\pi_j^h - \pi_j^l}{(1-\zeta)\pi_j^h + \zeta\pi_j^l}, \quad \text{for } i \leq j.$$

Some simple algebra shows that the above inequality is equivalent to $\pi_i^g \pi_j^l \leq \pi_i^l \pi_j^h$,which is exactly the monotone likelihood property satisfied by π^h and π^l. The same derivation also shows the necessity of the MLR property of π^h and π^l for $\pi^{G^c}(G^{c\prime}|G^c, e)$ to satisfy the same property.

To determine π^l and π^h, we assume that with $\bar{\zeta} = \mathbb{E}\zeta(e)$ evaluated at the ergodic distribution of effort, they can replicate the transition matrix of G^c without moral hazard π^{G^c}, namely:

$$\pi^{G^c} = \bar{\zeta}\pi^l + (1-\bar{\zeta})\pi^h,$$

subject to the requirement that π^h and π^l satisfy the MLR property. It is clear that the value of $\bar{\zeta}$ is just a normalization and we set it to be $\bar{\zeta} = 0.5$. Finally, we have put the maximum possible weight on the low G^c state for π^h and the opposite for π^l, subject to the restriction that the matrices still replicate π^{G^c} at $\bar{\zeta}$ and satisfy the MLR property. Given the parameterization of π^{G^c} with no moral hazard, and the value of the parameters $\phi = 0.965$ and $\varpi = 0.015$, the resulting matrices are:

$$\pi^h = \begin{bmatrix} 2\phi - 1 & \frac{4}{3}(1-\phi) & \frac{2}{3}(1-\phi) \\ 0 & 2(\phi + 2\varpi) - 1 & 2(1 - \phi - 2\varpi) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.93 & 0.0466 & 0.0234 \\ 0 & 0.99 & 0.01 \\ 0 & 0 & 1 \end{bmatrix}, \tag{D.1}$$

$$\pi^l = \begin{bmatrix} 1 & 0 & 0 \\ 4\varpi & 1 - 4\varpi & 0 \\ 2\varpi & 2(1 - \varpi - \eta) & 2\varpi - 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.06 & 0.94 & 0 \\ 0.03 & 0.04 & 0.93 \end{bmatrix}. \tag{D.2}$$

E Solution Method

E.1 The Solution of the IMD Economy

In what follows, we describe the computational algorithm to solve for the IMD model with moral hazard. Throughout this section, let $G = G^c + G^d$.

Solving for the labor supply For given (s, b) and b', we can solve for the optimal labor from the optimality condition. If the borrower chooses not to default, the optimal labor supply n^* solves:

$$h(n) \equiv (\theta n^\alpha - \chi)n^{1-\alpha} - \vartheta(1-n)^\sigma = 0$$

where $\vartheta = (\theta\alpha)/\gamma > 0$ and $\chi = G - (1 - \delta + \delta\kappa)b + q(s, b')(b' - \delta b)$. Since $h(1) = (\theta - \chi)$ and $h(0) = -\vartheta < 0$, there exists an $n^* \in (0, 1)$ such that $h(n^*) = 0$ and $c^* > 0$ if and only if $\theta - \chi > 0$. It is easy to show that n^* is unique. If the borrower chooses to default, we can use the same condition with $\vartheta = \theta^p\alpha/\gamma$ and $\chi = G$.

In what follows, we denote by $N_{\text{nd}}(s, b, b')$ the optimal labor supply in the case of no default, given the current state (s, b) and the bond choice for the next period b'; and we use $N_{\text{d}}(s)$ to denote the optimal labor supply in the case of default. Here we have chosen to suppress the dependence of N_{nd} on the bond price $q(s, b')$ for two reasons: first, given any pricing function $q(\cdot)$, the specific value of the bond price is determined by (s, b'); and second, to enhance computational efficiency, we will rewrite $N_{\text{nd}}(\cdot)$ as a function of θ and χ, where χ summarizes all the dependence of N_{nd} on G, b, b', and $q(s, b')$.

Solving for effort Before describing the solution of the Bellman equation, we discuss the calculation of optimal effort e given (s, b'), which is then used for evaluating the expected continuation value and updating the bond pricing function. In general, the optimality conditions for effort in are nonlinear in e and need to be solved for each combination of (s, b'). This would add considerable computational burden to the value function iteration. However, we have simplified the problem substantively by a careful choice of $v(e)$ and $\zeta(e)$ described in the calibration section.

Consider the case of no-default first. Given the value function from the previous iteration $V^b(s, b; k)$, in iteration $k + 1$, the optimal effort $e(s, b')$ without default solves the following condition:

$$\omega e = \beta(1-e)\sum_{s'} \pi^\theta(\theta'|\theta)(\pi^h(G'|G) - \pi^l(G'|G))V^{bi}(b', s'; k),$$

given s and candidate debt choice b'. The left hand side (LHS) equals to 0 for $e = 0$, and the right hand side (RHS) equals to 0 for $e = 1$. Since the RHS is always positive except for $e = 1$, it follows that $e(s, b')$ is between 0 and 1, and directly given by

$$e(s, b') = \frac{\beta\Delta^b(s, b')}{\omega + \beta\Delta^b(s, b')}, \tag{E.1}$$

where

$$\Delta^b(s, b') = \sum_{s'} \pi^\theta(\theta'|\theta)(\pi^h(G'|G) - \pi^l(G'|G))V^{bi}(b', s'; k) > 0$$

Given $e(s, b')$, we can simply evaluate the expected continuation value as

$$\mathbb{E}[V^{bi}(s', b'(s, b'); k] = \zeta \mathbb{E}^l[V^{bi}(s', b'; k)|s] + (1 - \zeta)\mathbb{E}^h[V^{bi}(s', b'; k)|s],$$

with $\zeta = (e(s, b') - 1)^2$. Similarly, for the case of default, the FOC of e in iteration $k + 1$ becomes

$$\omega e = \beta(1 - e) \sum_{s'} \pi^\theta(\theta'|\theta)(\pi^h(G'|G) - \pi^l(G'|G))[(1 - \lambda)V^a(s'; k) + \lambda V^{bi}(s', 0; k)],$$

so that the optimal effort $e^a(s)$ is again interior and given by

$$e^a(s) = \frac{\beta \Delta^a(s)}{\omega + \beta \Delta^a(s)}, \tag{E.2}$$

where

$$\Delta^a(s) = \sum_{s'} \pi^\theta(\theta'|\theta)(\pi^h(G'|G) - \pi^l(G'|G))[(1 - \lambda)V^a(s'; k) + \lambda V^{bi}(s', 0; k)]]$$

Accordingly, the continuation value equals to

$$\begin{aligned}\mathbb{E}[V^a(s'(s)] = \zeta \mathbb{E}^l[(1 - \lambda)V^a(s'; k) + \lambda V^{bi}(s', 0; k)|s] \\ + (1 - \zeta)\mathbb{E}^h[(1 - \lambda)V^a(s'; k) + \lambda V^{bi}(s', 0; k)|s],\end{aligned}$$

with $\zeta = (e^a(s) - 1)^2$.

Solving the Bellman Equation To find a solution to the model, we combine equations (12)–(14) as well as the pricing equation in (16) into one Bellman equation of four functions: three value functions and one pricing function. We can then use backward induction to solve the functional equation. More precisely, let $V^{bi}(b,\ s; k-1)$, $V_n^{bi}(b, s; k-1)$, $V^a(s; k-1)\}$ and $q(s, b'; k-1)$ denote the value and pricing functions obtained in the k'th iteration. We first solve:

$$\begin{aligned}V_n^{bi}(b, s; k) = \max_{c,n,b'} U(c, N_{\text{nd}}(s, b, b'; k), e^b(s, b')) + \beta \mathbb{E}[V^{bi}(s', b'(s, b'); k - 1] \\ \text{s.t. } c + G + q(s, b; k)(b' - \delta b) \le \theta f(N_{\text{nd}}(s, b, b'; k - 1)) + (1 - \delta + \delta\kappa)b,\end{aligned}$$

where,

$$V^{bi}(b,\ s; k) = \max\left\{V_n^{bi}(b,\ s; k), V^a(s; k)\right\},$$

and

$$V^{a}(s;k) = u(\theta^{p}(\theta)f(N_{\mathrm{d}}(s)) - G) + h(1 - N_{\mathrm{d}}(s)) - v(e^{a}(s))$$
$$+ \beta\mathbb{E}[(1-\lambda)V^{a}(s';k-1) + \lambda V^{bi}(s',0;k-1)]$$

As explained earlier, we denote the labor supply function in the no default case by $N_{\mathrm{nd}}(s,b,b';k)$ to make explicit the dependence of $N_{\mathrm{nd}}(\cdot)$ on the bond pricing function $q(\cdot;k)$ in each iteration. This is a standard dynamic programming problem that delivers value and policy functions for consumption, labor and bond choices, as well as default decisions. Once we have these, we can update the pricing function via:

$$q^{x}(s,b';k+1) = \mathbb{E}^{x}\left[(1 - D(s',b';k))\frac{(1-\delta) + \delta[\kappa + q(s', B(s',b';k);k)]}{1+r}\middle| s\right]$$

for $x = h, l$, where $D(s,b;k)$ and $b(s,b;k)$ are the default and bond holding decisions obtained in iteration k. In general, this shows that $q(\cdot;k)$ is obtained in iteration $k-1$.

To implement the backward induction algorithm, we use discrete space value function iteration. Since (θ, G^{c}) is discrete by assumption, we only need to discretize G^{d} and b. In particular, we set G^{d} to be equally spaced over $[-\bar{m},\bar{m}]$ with N_{d} grid points, and with equal probability on each grid point for simplicity. Moreover, we discretize the bond holding space $\mathcal{B}$ with N_{b} grid points. We iterate on the value function and the pricing function on the discretized space $\Theta \times \mathcal{G}^{c} \times \mathcal{G}^{d} \times \mathcal{B}$ until convergece, namely, until

$$\max|V^{bi}(s,b;k) - V^{bi}(s,b;k+1)| \text{ and } \max|q(s,b';k) - q(s,b';k+1)|$$

are both smaller than some convergence criterion. Moreover, we use two parameters $\zeta_{V}, \zeta_{q} \in [0,1]$ to control the updating speed of $V^{bi}(\cdot)$ and $q(\cdot)$. Setting $\zeta_{q} > 0$ is useful for the convergence of $q(\cdot)$ as well.

Note that it is important to have a continuously distributed G^{d} to smooth off discrete changes in the default decision $D(s,b)$ and enhance the convergence properties of the model. In principle, we could keep G^{d} as a continuous state variable in the computation, and use the involved procedure of Chatterjee and Eyigungor (2012) to obtain the functions $D(\cdot, G^{d}, \cdot)$ and $b(\cdot, G^{d}, \cdot)$ accurately. Instead, we use a discrete approximation of G^{d}, which is straightforward to implement, and we find that such an approximation works good enough to improve the convergence properties of the algorithm to compute our model.

E.2 The Solution of the FSF Economy

Using the functional forms for $v(e)$ and the system of equations that characterizes the solution for the Fund can be rewritten as:

$$c(x,s) = \frac{1+\nu_{b}(x,s)}{1+\nu_{l}(x,s)}x$$

$$c(x,s)\gamma(1-n(x,s))^{-\sigma} = \theta\alpha n(x,s)^{\alpha-1},$$

$$x(s') = \frac{1+\nu_b(x,s)+\varphi(x,s')}{1+\nu_l(x,s)}\eta x = \frac{1+\nu_b(x,s)+\tilde{\xi}\frac{2(1-e)[\pi^h(G'|G)-\pi^l(G'|G)]}{\pi(G'|G,e)}}{1+\nu_l(x,s)}\eta x,$$

$$\varphi(x,s') = \tilde{\xi}(s)\frac{2(1-e)[\pi^h(G'|G)-\pi^l(G'|G)]}{\pi^G(G'|G,e)} \text{ where } \tilde{\xi}(s) = \frac{\xi(s)}{\mu_b(s)}$$

$$(1+r)\tilde{\xi}(x,s)\omega = \sum_{s'|s}\pi^\theta(\theta'|\theta)[\pi^h(G'|G)-\pi^l(G'|G)] \times \left[\frac{1+\nu_l(x,s)}{x}(1-e)V^{lf}(x',s') - \eta\tilde{\xi}(x,s)V^{bf}(x',s')\right]$$

$$\omega e = \beta(1-e)\sum_{G',\theta'}[\pi^h(G'|G)-\pi^l(G'|G)]\pi^\theta(\theta'|\theta)V^{bf}(x',s'),$$

where

$$V^{bf}(x,s) = \log(c(x,s)) + \frac{\gamma(1-n(x,s))^{1-\sigma}}{1-\sigma} - \omega e^2(x,s) + \beta\sum_{s'}\pi^G(G'|G,e(x,s))\pi^\theta(\theta'|\theta)V^{bf}(x(s'),s').$$

$$V^{lf}(x,s) = \theta n(x,s)^\alpha - c(x,s) - G + \frac{1}{1+r}\sum_{s'}\pi^G(G'|G,e(x,s))\pi^\theta(\theta'|\theta)V^{lf}(x(s'),s').$$

The solution to this system of equations is found numerically using a policy iteration algorithm but we simplify our computations by rewriting the system as a function of (z,s), where $z = \frac{x}{\eta}$. Note that we are abusing notation by using the same name for the policy functions that depend on (z,s). If we do this, we obtain:

$$c(z,s) = \frac{1+\nu_b(z,s)}{1+\nu_l(z,s)}\eta z$$

$$c(z,s)\gamma\left(1-n(z,s)\right)^{-\sigma} = \theta\alpha n(z,s)^{\alpha-1},$$

$$z'(z,s) = \frac{1+\nu_b(z,s)}{1+\nu_l(z,s)}\eta z + \xi(z,s)\frac{2(1-e)[\pi^h(G'|G)-\pi^l(G'|G)]}{\pi(G'|G,e)}$$

$$\omega e = \beta(1-e)\sum_{G',\theta'}[\pi^h(G'|G)-\pi^l(G'|G)]\pi^\theta(\theta'|\theta)V^{bf}(z',s')$$

$$(1+r)\,\xi(z,s)\omega = \sum_{s'|s}\pi^\theta(\theta'|\theta)[\pi^h(G'|G)-\pi^l(G'|G)][(1-e)V^{lf}(z',s') - \xi(z,s)\eta V^{bf}(z',s')]$$

where $\xi(z,s) = \frac{\tilde{\xi}(z,s)\eta z}{1+\nu_l(z,s)}$. With this normalization, $\nu_l(z,s)$ does not appear in the optimality condition for effort, which simplifies our computations. To solve the system of equations above, we proceed as follows.

Since the shocks are already discrete, we discretize the relative Pareto weight z. For each shock s, we first determine the points l and m such that, at all gridpoints below z_l, the participation constraint (PC) of the borrower is binding and the solution is the same, while at all gridpoints above z_m, the PC of the lender is binding and the solution is the same. At all points between z_{l+1} and z_{m-1}, the solution is the full commitment solution, since the PC does not bind for the borrower or the lender. To determine l and m, we follow the steps below.

To determine l, for a given shock $s = (\theta, G)$, we check first if the participation constraint of the borrower is binding at the last (highest) Pareto weight for the borrower z_N. If yes, we set $l = N$. If not, we find z_l such that the agent is indifferent between staying in the contract or defaulting, namely:

$$U(c(s, z_l), n(s, z_l), e(s, z_l)) + \beta \sum_{G', \theta'} \pi^\theta(\theta'|\theta)\pi^G(G'|G, e)V^{bf}(z', s') = V^a(s) \tag{E.3}$$

To do this, for a candidate z, we first find the optimal level of effort using the following condition:

$$\eta z \gamma (1 - n)^{-\sigma} = \theta \alpha n^{\alpha - 1} \tag{E.4}$$

where we have set $c = \eta z$ since $\nu_l(z, s) = \nu_b(z, s) = 0$ at the point of indifference for the borrower. Next, we find the closest point on the grid z_l such that $z_l \simeq z$. We now find the optimal effort as follows. First, use the initial guess for the multiplier $\xi_0(s, z_l)$ and for each candidate for effort e calculate $z'(G', z, s)$ as follows:

$$z'(G', z, s) = \eta z_l + \xi_0(s, z_l) \frac{2(1 - e)[\pi^h(G'|G) - \pi^l(G'|G)]}{(e - 1)^2 \pi^l(G'|G) + (1 - (e - 1)^2)\pi^h(G'|G)} \tag{E.5}$$

Then find the level of effort e that satisfies the ICE condition:

$$\omega e = \beta(1 - e) \sum_{G', \theta'} [\pi^h(G'|G) - \pi^l(G'|G)]\pi^\theta(\theta'|\theta)V^{bf}(z'(G', z, s), s') \tag{E.6}$$

and if $\beta(1 - e) \sum_{G', \theta'} [\pi^h(G'|G) - \pi^l(G'|G)]\pi^\theta(\theta'|\theta)V^{bf}(z'(G', z, s), s') - \omega e < 0$ set $e = 0$.

Once we have the optimal effort, we can find the new multiplier $\xi_1(s, z_l)$ from the FOC for effort:

$$\begin{aligned} 0 = (1 - e) &\sum_{G', \theta'} [\pi^h(G'|G) - \pi^l(G'|G)]\pi^\theta(\theta'|\theta)V^{lf}(z'(G', z, s), s') \\ &+ \xi_1(s, z_l)\Big[-\eta \sum_{G', \theta'} [\pi^h(G'|G) - \pi^l(G'|G)]\pi^\theta(\theta'|\theta)V^{bf}(z'(G', z, s), s') - \omega(1 + r)\Big] \end{aligned} \tag{E.7}$$

With the optimal levels of consumption, effort, labor and future Pareto weight, we then iterate on equation (E.3) to find the actual Pareto weight z for which it is satisfied exactly. This determines l as the index of the closest point in the grid to z and we then set:

$$e(s, z_1 : z_l) = e(s, z_l), \;\; c(s, z_1 : z_l) = c(s, z_l), \;\; n(s, z_1 : z_l) = n(s, z_l)$$

$$
\begin{aligned}
V^{bf}(s, z_1 : z_l) &= V^{af}(s),\ \xi(s, z_1 : z_l) = \xi_1(s, z_l) \\
z'(G', z_1 : z_l, s) &= \eta z_l \\
&\quad + \xi_1(s, z_l) \frac{2\big[1 - e(s, z_l)\big]\big[\pi^g(G'|G) - \pi^b(G'|G)\big]}{\big[e(s, z_l) - 1\big]^2 \pi^b(G'|G) + \big[1 - (e(s, z_l) - 1)^2\big]\pi^g(G'|G)} \\
V^{lf}(s, z_1 : z_l) &= \theta n(s, z_l)^\alpha - c(s, z_l) - G \\
&\quad + \frac{1}{1+r} \sum_{G', \theta'} \pi^\theta(\theta'|\theta)\pi^G(G'|G, e)V^{lf}(z'(G', z_l, s), s')
\end{aligned}
$$

We follow a similar procedure to find m. For a given shock $s = (\theta, G)$, check first if the participation constraint of the lender is binding at the first (lowest) Pareto weight z_1 for the borrower. If yes, we set $m = 1$. If not, we use fzero to find z_m such that the lender is indifferent between staying in the contract or defaulting, namely:

$$
\theta n(s, n_m)^\alpha - c(s, n_l) - G + \frac{1}{1+r} \sum_{G', \theta'} \pi^\theta(\theta'|\theta)\pi^G(G'|G, e)V^{lf}(z'(G', z, s), s') = Z. \tag{E.8}
$$

To do this, for a candidate z, we first find the optimal level of effort using (E.4), where we set $c = \eta z$ since $\nu_l(z, s) = \nu_b(z, s) = 0$ at the point of indifference for the lender. Next, we find the closest point on the grid z_m such that $z_m \simeq z$. We now find the optimal effort as follows. First, use the initial guess for the multiplier $\xi_0(s, z_m)$ and for each candidate for effort e calculate $z'(G', z, s)$ using (E.5). Then find the level of effort e that satisfies the ICE condition (3). Once we have the optimal effort, we can find the new multiplier $\xi_1(s, z_m)$ from the FOC for effort (E.7).

With the optimal levels of consumption, effort, labor and future Pareto weight, we then iterate on equation (E.8) to find the initial Pareto weight z for which this is satisfied exactly. This determines m as the index of the closest point in the grid to z and we then set:

$$
\begin{aligned}
e(s, z_m : z_N) &= e(s, z_m),\ c(s, z_m : z_N) = c(s, z_m),\ n(s, z_m : z_N) = n(s, z_m) \\
V^{lf}(s, z_m : z_N) &= Z,\ \xi(s, z_m : z_N) = \xi_1(s, z_m) \\
z'(G', z_m : z_N, s) &= \eta z_m \\
&\quad + \xi_1(s, z_m) \frac{2(1 - e(s, z_m))\big[\pi^g(G'|G) - \pi^b(G'|G)\big]}{\big[e(s, z_m) - 1\big]^2 \pi^b(G'|G) + \big[1 - (e(s, z_m) - 1)^2\big]\pi^g(G'|G)} \\
V^{bf}(s, z_1 : z_l) &= U(c(s, z_m), n(s, z_m), e(s, z_m)) \\
&\quad + \beta \sum_{G', \theta'} \pi^\theta(\theta'|\theta)\pi^G(G'|G, e)V^{bf}(z'(G', z_m, s), s')
\end{aligned}
$$

Once we have determined the solution for $z_1 : z_l$ and $z_m : z_N$, the consumption and labor for the points in between is set to the full commitment (equal to the first best solution):

$$
c(s, z_{l+1} : z_{m-1}) = c_{fb}(s, z_{l+1} : z_{m-1})
$$

$$n(s, z_{l+1} : z_{m-1}) = n_{fb}(s, z_{l+1} : z_{m-1})$$

For each of these gridpoints, we solve for effort and the multiplier as above, namely, we first use the initial guess for the multiplier $\xi_0(s, z_m)$ and for each candidate for effort e calculate $z'(G', z, s)$ using (E.5). Then find the level of effort e that satisfies the ICE condition (3) and once we have the optimal effort, we can find the new multiplier $\xi_1(s, z_m)$ from the FOC for effort (E.7).

One way to speed up the code is to use the guess for effort from the previous iteration $e_0(z, s)$ to calculate $z'(G', z, s)$:

$$z'(G', z, s) = \eta z_l + \xi_0(s, z_l) \frac{2\left[1 - e_0(s, z)\right]\left[\pi^h(G'|G) - \pi^b(G'|G)\right]}{\left[e_0(s, z) - 1\right]^2 \pi^b(G'|G) + \left[1 - (e_0(s, z) - 1)^2\right]\pi^g(G'|G)}$$

We can then use the IC condition (E.6) to solve for effort without using a non linear equation solver:

$$e_1(s, z) = \frac{\beta \sum_{G',\theta'} \left[\pi^g(G'|G) - \pi^b(G'|G)\right]\pi^\theta(\theta'|\theta)V^{bf}(z'(G', z, s), s')}{\omega + \beta \sum_{G',\theta'} \left[\pi^g(G'|G) - \pi^b(G'|G)\right]\pi^\theta(\theta'|\theta)V^{bf}(z'(G', z, s), s')},$$

and we also iterate on effort until convergence.

Finally, we use the allocations to compute the asset prices $q(s'|s)$ and $q(s) = \sum_{s' \in S} q(s'|s)$ as well as the asset holdings for the borrower and lender as defined in Section 3.

F Welfare Analysis

This section describes how we compute the welfare numbers that appear in Tables 5 and 6 in the main text.

F.1 Welfare Gains

The welfare gains of the Fund displayed in Table 5 are computed in a standard techniques to reflect them in consumption-equivalent terms. To do this let the value of the borrower from a sequence of allocations $\{c(s^t), n(s^t), e(s^t)\}$ starting from a particular initial state at $t = 0$ be denoted by:

$$V^{bi}(\{c(s^t), n(s^t), e(s^t)\}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c(s^t), n(s^t), e(s^t))$$

where

$$U(c, n, e) = \log(c) + \gamma \frac{(1-n)^{\sigma_n} - 1}{1 - \sigma_n} - \omega e^2$$

is the period utility function. Denote the allocations in the fund by $\{c^f(s^t), n^f(s^t), e^f(s^t)\}$ and the allocations in the IMD economy by $\{c(s^t), n(s^t), e(s^t)\}$. Given our recursive formulation, the value for the borrower in the IMD and the Fund economies are equal to:

$$V^{bi}(s, a) \quad = \quad V^{bi}(\{c(s^t), n(s^t), e(s^t)\})$$

$$V^{bf}(s,a) \quad = \quad V^{bf}(\{c^f(s^t), n^f(s^t), e^f(s^t)\})$$

respectively, where a is the debt position of the borrower and (s, a) is the initial state, with the corresponding policy functions for c, n, and e in the two economies. Note that we can set the domain for a to be identical in both economies. Moreover, although the endogenously determined asset (debt) limits differ in the two economies and over the state s, the value functions $V^{bi}(s, a)$ and $V^{bf}(s, a)$ can always be extended beyond the limit by using the corresponding autarky values.

To define the consumption-equivalent welfare gain of the Fund $\chi(s, a)$ that is displayed in Table 5, we let

$$V^{bi}(s,a;\chi) = V^{bi}(\{(1+\chi)c(s^t), n(s^t), e(s^t)\})$$

and we define the consumption-equivalent welfare gain of the Fund versus the IMD economy by the following condition:

$$V^{bi}(s,a;\chi) = V^{bf}(s,a)$$

which, given the functional form of $U(c, n, e)$, satisfies:

$$\frac{\log(1+\chi)}{1-\beta} + V^{bi}(s,a) = V^{bf}(s,a)$$

From the previous condition, it follows that:

$$\chi(s,a) = \exp\left[(V^{bf}(s,a) - V^{bi}(s,a))(1-\beta)\right] - 1. \tag{F.1}$$

F.2 Welfare Decomposition

As stated in the main text, we decompose the welfare gains of the Fund by calculating the percentage of $\chi(s, a)$ that can be attributed to the following four factors: (i) no productivity penalty; (ii) no debt market exclusion; (iii) a higher debt capacity; and (iv) the presence of state contingency (insurance). Since it is not obvious how to isolate the four different factors, we resort to the following simulation exercises to compute the contribution of each factor.

No Productivity Penalty To compute the gain that can be attributed to not having a productivity penalty in the Fund, we fix the initial state (s, a) and use the policy functions from the IMD economy to do many simulations of the IMD economy for a long enough number of periods without imposing the penalty when the country defaults. For each simulation, we then compute the value of the borrower using the discounted present value of the period utility and we take the average across all simulations. We denote the value function for the borrower obtained this way by $V_1^b(s, a)$.

No Debt Market Exclusion To compute the gain that can be attributed to not being excluded from financial markets, we conduct the same simulations as above but we set the number of exclusion periods to zero. We denote the value function for the borrower obtained this way by $V_2^b(s,a)$.

Bigger Debt Capacity The exercise to assess the contribution to welfare of a higher debt capacity in the Fund is more involved. Recall that the bond component $a'(s,a)$ in the Fund is constructed from the state-contingent asset choices $a(s',a)$, which are subject to state-contingent borrowing limits $A(s')$. As a result, it is not straightfoward to pin down the allocations associated with the bond policy function $a'(s,a)$ alone. To partially address this, we solve the following problem for the borrower:

$$V_m^b(s,a) = \max_{c,n,a',e} U(c, 1-n) - v(e) + \beta \mathbb{E}\left[V_m^b(s',a')\big|s,e\right] \tag{F.2}$$
$$\text{s.t.} c + q\left(a' - \delta a\right) = \theta n^\alpha - G + (1-\delta+\delta\kappa)a,$$
$$a' \geq A(\bar{s}),$$

where the bond price is set at the risk free long term bond price q and the borrowing limit is fixed at the endogenous borrowing limit in the Fund $A(\bar{s})$ corresponding to the shock state $\bar{s}$. Note that this recursive problem corresponds to an incomplete market (IM) setup without default and with a particular borrowing limit. We denote the value of the borrower from this problem as $V_3^b(s,a) = V_m^b(s,a)$, and we identify the welfare gains associated with V_m^b and the corresponding bond policy function a' as a proxy to the welfare contribution of the bond component from $a'(s,a)$ in the Fund for the particular state $\bar{s}$.

If we denote the value of the borrower in the IMD economy by $V_0^b = V^{bi}(s,a)$ and the value of the borrower in the fund by $V_4^b = V^{bf}(s,a)$, we can decompose the welfare gain $\chi(s,a)$ into the following four components $\chi_i(s,a)$ using V_i^b for $i = 1,\ldots,4$. In particular, we have that[21]:

$$1 + \chi(s,a) = \big(1+\chi_1(s,a)\big) \times \cdots \times \big(1+\chi_4(s,a)\big)$$

In addition, to evaluate the *percentage* contribution of each χ_i to the overall χ, we define:

$$\begin{aligned}\pi_i(s,a) &= \frac{\big(1+\chi_1(s,a)\big)\cdots\big(1+\chi_2(s,a)\big) - \big(1+\chi_1(s,a)\big)\cdots\big(1+\chi_{i-1}(s,a)\big)}{\chi(s,a)} \\ &= \frac{\big(1+\chi_1(s,a)\big)\cdots\big(1+\chi_{i-1}(s,a)\big)\chi_i(s,a)}{\chi(s,a)},\end{aligned} \tag{F.3}$$

for $i = 2,\ldots,4$, with $\pi_1(s,a) = \chi_1(s,a)/\chi(s,a)$.

[21] For consistency, since we construct V_1^b and V_2^b by simulation, we also compute V_i^b for $i = 0,3,4$ by simulation for different combinations of (s,a).

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